This homework is due on April 25, by start of class, 12:50pm. Write each solution on a different sheet of paper, and put your name and student ID on each page. You can turn in the homework in the CS103 drop box in Gates, by emailing it to `cs103-spr1314-hw@lists.stanford.edu`, or turning it in in class.

Problem 1

Nim is a family of games played by two players. The game is set up with several piles of stones. Two players take turns removing stones from the piles, such that each move involves removing one or more stones from a single pile. The winner of the game is the player who removes the last stone from play. In other words, if there are no stones available at the start of a player's turn, they have lost the game.
Consider games of Nim which begin with two piles with an equal number of stones. Use induction to show that the player who plays second can always win such a game.

Problem 2

Consider the following recursive definition of a function $f : \mathbb{N} \to \mathbb{N}$.

$$
\begin{aligned}
f(1) &= 1 \\
f(n) &= 2f(n-1) + 1
\end{aligned}
$$

Find a non-recursive definition for f, and prove by induction that this definition is correct.

Problem 3

Prove using induction that $n! < n^n$ for $n > 1$.

Problem 4

Show that any wire whose length is exactly x inches ($x \in \mathbb{N}, x \geq 12$) can be cut into some number of pieces, each of which is exactly 4 inches or 5 inches long.

Problem 5

For this problem, a *polygon* is a flat, closed shape whose boundary is closed chain of line segments and that has at least 3 vertices. A *diagonal* of a polygon is a straight line joining two non-adjacent vertices of the polygon. A *convex polygon* is a polygon such that any diagonal lies in its interior. Prove by induction that a convex polygon with n vertices has at most $n-3$ non-intersecting diagonals. That is, prove that the cardinality of any set consisting only of non-intersecting diagonals of the polygon is at most $n-3$.
Hint : You may assume without proof that if P is a non-triangular convex polygon with n vertices, then a diagonal of P divides P into an x-vertex convex polygon and a y-vertex convex polygon, such that $n = x + y - 2$ and$x < n$ and $y < n$. (the two sub-polygons share exactly two vertices: the vertices of the dividing diagonal).
For this problem, your geometric reasoning about diagonals and their intersections (especially how a polygon's diagonals relate to those of its constituent sub-polygons) does not need to be as formal as the rest of your induction proof. While your reasoning needs to be correct, it does not need a lot of detail.