This homework is due on May 2, by start of class, 12:50pm. Write each solution on a different sheet of paper, and put your name and student ID on each page. You can turn in the homework in the CS103 drop box in Gates, by emailing it to `cs103-spr1314-hw@lists.stanford.edu`, or turning it in in class.

Problem 1

In what follows, if p is a polygon, then let $A(p)$ denote its area.

1. Define the relation $=_A$ over the set of all polygons as follows: if x and y are polygons, then $x =_A y$ if and only if $A(x) = A(y)$. Is $=_A$ an equivalence relation? If so, prove it. If not, prove why not.

2. Define the relation $\leq_A$ over the set of all polygons as follows: if x and y are polygons, then $x \leq_A y$ if and only if $A(x) \leq A(y)$. Is $\leq_A$ a partial order? If so, prove it. If not, prove why not.

Problem 2

Let $G = (V, E)$ be an undirected graph. The complement of G is the graph $G^c = (V, E')$, that has the same nodes but a different edge set E': for any nodes $u, v \in V$, the edge $(u, v) \in E'$ if and only if $(u, v) \notin E$. In other words, the edges in G^c are those not present in G and vice versa.

Prove that for every undirected graph G, at least one of G and G^c is connected. An undirected graph G is called connected if it contains a path between every pair of its vertices.

(Hint: To prove a statement of the form "P or Q," you can instead prove the statement "if P is false, then Q is true." Show that if G isn't connected, then G^c must be connected.)

Problem 3

A tournament graph is a directed graph with $n \geq 1$ nodes where there is exactly one edge between any pair of distinct nodes and there are no self-loops. Show that if a tournament graph contains a cycle, then it contains a cycle of length 3, that is, a cycle containing 3 edges.

(Hint: consider using a proof by extremal case: consider the smallest cycle in a tournament graph containing a cycle and proceed by contradiction to show that it must have length 3*)*

Problem 4

Let G be an undirected graph. The degree of a node v is the number of edges incident to v, i.e. the number of edges with v as one of their endpoints. Prove that G contains two nodes with the same degree.

(Hint: consider two cases: the case in which every vertex has degree ≥ 1 *and the case in which there is one or more vertices of degree* 0*.)*