This homework is due on May 9, by start of class, 12:50pm. Write each solution on a different sheet of paper, and put your name and student ID on each page. You can turn in the homework in the CS103 drop box in Gates, by emailing it to `cs103-spr1314-hw@lists.stanford.edu`, or turning it in in class.

Revised 5/5/2014: in Problem 2 you can use OR

Problem 1

State, with the help of truth tables, whether the following statements in propositional logic are valid or not:

1. $((p \to q) \wedge p) \to q$.
2. $((p \to q) \wedge \neg q) \to \neg p$.
3. $((p \to q) \wedge (p \to \neg q)) \to p$
4. $((p \vee q) \wedge \neg p) \to q$.

Problem 2

Find an equivalent statements for the following statements, obtained by converting all implications into their equivalences containing solely $\neg$, $\vee$ and $\wedge$. Ensure that the final result should not have any negations except for direct negations of predicates:

1. $(p \to q) \to r$
2. $(p \wedge q) \to p$
3. $\neg p \leftrightarrow q$
4. $(p \leftrightarrow q) \wedge p$

Problem 3

Find the negations of the following first order logic statements. The final form should not have any negations except for direct negations of predicates.

1. $\forall p.\forall q.(isOdd(p) \wedge isOdd(q) \rightarrow isOdd(p+q))$
2. $\exists S.(Set(S) \wedge \forall x.x \notin S)$

Problem 4

Formalize the english statement using first order logic using the list of first order predicates and functions provided. You can use any first order construct (equality, connectives, quantifiers etc.) but you must only use the predicates, functions and constants provided:

1. Given the predicate

   $Natural(x)$, which states that x is a natural number,

   the function

   $Product(x, y)$, which yeilds the product of x and y,

   and the constants 1 and 7, write a statement in first order logic which says "7 is prime".

2. Given the predicates

   $Morality(x)$ which states that x is a morality,
   $Practiced(x)$ which states that x is practiced, and
   $Preached(x)$ which states that x is preached,

   write a statement in first order logic which states "there are exactly two moralities; one of which is practiced but not preached, and one of which is preached but not practiced" (paraphrased from a quote by Bertrand Russel).

Problem 5

For each of the languages over the indicated alphabets, construct a DFA which accepts precisely those strings that are in the language. Specify the DFA as a state transition diagram:
We have an online tool that can be used to design, test and submit DFAs for this question. To use it, visit `https://www.stanford.edu/class/cs103/cgi-bin/nfa/edit.php`. We strongly recommend this tool, as it makes it easy to design, test and submit your solutions. If you submit it via this system, please make a note of it in your homework submission so that we know to look online for your answers.

1. For the alphabet $\sum = \{\mathit{0, 1, 2}\}$, construct a DFA for the language L= $\{w \in \sum^* \mid$ w contains exactly two *2* s$\}$

2. For the alphabet $\sum = \{a, b, c...z\}$, construct a DFA for the language L= $\{w \in \sum^* \mid$ w contains the word "*cocoa*" as a substring$\}$. As a shorthand, you can specify multiple letters in a transition by using set operations on $\sum$ (for example $\sum$ - $\{a, b\}$).
(Hint: Here, notice that the word "cocoa" contains a consecutive and repeating "co", which creates a tricky situation. Specifically, the DFA needs to be able to handle strings which contain more than two consecutive "co"s)