This homework is due on May 30, by start of class, 12:50pm. Write each solution on a different sheet of paper, and put your name and student ID on each page. You can turn in the homework in the CS103 drop box in Gates, by emailing it to `cs103-spr1314-hw@lists.stanford.edu`, or turning it in in class.

Problem 1 (20 points)

Let $\Sigma = \{0, 1\}$. Draw the state transition diagram for a Turing machine whose language is $L = \{w \in \Sigma^* | w \text{ contains } 01 \text{ as a substring}\}$.

Problem 2 (45 points)

1. Prove that there exists a Turing machine M such that the language L recognized by M is decidable, but M is not a decider. This shows that just because a Turing machine's language is decidable, it's not necessarily the case that the Turing machine itself must be a decider.

2. Prove that for every language L, there is a decider M that accepts every string in L and another decider M' that rejects every string not in L. Explain why this doesn't prove that every language is decidable.

3. Find a pair of languages A and B such that A is a subset of B, B is decidable, but A is not decidable. This shows that a subset of a decidable language is not necessarily decidable, i.e. bigger languages are not necessarily 'harder.'

Problem 3 (35 points)

Let $HALT$ be the language $\{\langle M, w\rangle : M \text{ is a TM that halts on } w\ \}$. Let $ALLHALT$ be the language $\{\langle M\rangle : M \text{ is a TM that halts on all inputs } \}$. Use a reduction from $HALT$ to show that $ALLHALT$ is not decidable.