CS103 Practice Midterm

Problem 1

For each of the following statements about finite sets, either prove that they are true or give a counterexample to show that they are false.

(i) For any two sets A and B, $B \setminus (B \setminus A) = A$. Here $B \setminus A$ refers to set difference, $B \setminus A = \{x : x \in B \text{ and } x \notin A\}$

(ii) If A, B are sets, then $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$

(iii) For sets A, B, C, if $A \subseteq B$ and $B \nsubseteq C$, then $A \nsubseteq C$.

(iv) For sets A, B, we have $(A \setminus B) \cup (A \cap B) = A$.

Problem 2

Prove by induction that if a set S contains n elements, where $n \geq 0$, then its power set $\mathcal{P}(S)$ contains 2^n elements.

Problem 3

Show that if an undirected graph G has n vertices, each of degree at least $(n-1)/2$, then the graph is connected.

Problem 4

State which of the following are equivalence relations, and which are partial orders.

(i) Let $xRy = \{(x, y) \in \mathbb{N} \times \mathbb{N} : x|y\}$, where $x|y$ means that x is a factor of y.

(ii) Let S be the set of strings (or sequences of characters, like "cat"). Let $aRb = \{(a, b) \in S \times S : a \text{ and } b \text{ have the same length }\}$.

(iii) Let $xRy = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : |x - y| \leq 1\}$.

Problem 5

Suppose $\sim$ is a relation on a set A, and that $\sim$ is reflexive and for all $a, b, c \in A$, if $a \sim b$ and $a \sim c$, then $b \sim c$. Show that $\sim$ is an equivalence relation.