Practice Final

Problem 1

Prove using induction that every chess board of size $2^n \times 2^n$, with a single corner tile removed, can be covered using triominoes, the L- shaped tiles defined in HW 1. Consider $n \in \mathbb{N}$, $n>0$.

Problem 2

Let $G = (V, E)$ be an undirected graph with no self loops. Prove that if the degree of every node in G is at least $^{|V|}/_2$, then G is connected.

Problem 3

Construct a DFA to the language $L = \{s | s \textit{ represents a binary number divisible by 7}\}$. The alphabet is $\sum = \{0, 1\}$.

Problem 4

Let $\sum = \{0, 1\}$, and define the language $L = \{00^*ww00^* | w \in \sum^*\}$. Prove that L is not regular.

Problem 5

Let $A_{TM} = \{<M, w> | M \textit{ is a TM and } M \textit{ accepts } w\}$.
Using the fact that the language A_{TM} is undecidable, prove that the language

$$L_{101} = \{<M> | L(M) \textit{ contains the string "101"}\}$$

is undecidable.

Problem 6

Let $L_l = \{<M, w>|M \textit{ moves its head left at least once when operated on input } w\}$. Can you prove that L_l is undecidable using a proof technique similar to the one used in the previous problem? Prove it if you can, and if not, explain why.