Write every problem on a separate sheet of paper, write *legibly*, and write your name, your student ID, this Homework # and the problem # on top of each page.

The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.)

Make sure you are familiar with the collaboration policy, and read the overview in the class homepage theory.cs.stanford.edu/∼trevisan/cs154.

Recall that our late policy is that late homework is not accepted (however the lowest homework grade is dropped).

Refer to the class home page for instructions on how to submit the homework.

Homework 7

Due on Thursday, March 8, 2012, 9:30am

1. (4 points) Given a collection C of sets over a universe U, a *one-hitter* is defined to be a set $H \subseteq U$ such that each set in C shares exactly one member with H. For example, $H = \{2, 3\}$ is a one-hitter for the collection $C = \{\{1, 2, 4\}, \{3, 4, 5\}, \{1, 2, 5, 6\}\}$, because H "hits" each set in C exactly once.

   The ONE-HITTER problem is: given a collection of sets C and an integer k, does a one-hitter H for C exist where $|H| = k$?

   Show that this problem is NP-complete.

2. (3 points) Here we will look at a variant on the Minesweeper game, and show that it is NP-complete to solve. (Moral: Hardcore gaming could quite possibly save the world.)

   In our variant, we have an undirected graph G where every node is either labelled with a mine, or is labelled with a number indicating how many neighbors of that node are labelled with mines. Initially, all node labels are hidden. The game progresses by selecting nodes one at a time to *uncover*. If a node labelled with a mine is uncovered, the game ends in a loss. Otherwise, the node's label is revealed and the game continues. The game is won once all nodes not labelled with a mine have been uncovered.

The following problem captures the notion of a valid state for our Minesweeper variant. We define this problem, MINESWEEP, as follows. Given:

- a graph $G = (V, E)$,
- a set of vertices $S \subseteq V$ that have been uncovered,
- a function $L : S \to \{1, ..., |V|\}$ which gives the number of mines adjacent to each uncovered vertex, and
- an integer m

Determine if there is a placement of m mine labels on the vertices in $V - S$ such that for every $v \in S$, v has exactly $L(v)$ mines as neighbors.

Prove that MINESWEEP is NP-complete.

Hint: You may use the results of the previous problem in your proof here.

3. (3 points) For each of the following, determine whether such a problem is: known to exist, known not to exist, or neither. If you answer "neither", explain why discovering such a problem would make you rich. Otherwise, briefly justify your answer.

   (a) An NP-complete problem which can be solved in exponential time.

   (b) An NP-complete problem which cannot be solved in exponential time.

   (c) An NP problem which is not regular and is not NP-complete.