Practice Midterm

Will not be graded.

Problems

1. Let $\Sigma = \{0,1\}$ and for all $n \geq 0$, define $L_n = (\{0,1\}^*0\{0,1\}^n0) \cup (\{0,1\}^*1\{0,1\}^n1)$.

   Show how to construct a DFA that accepts L_n.

2. Let $\Sigma = \{1,0\}$ and for all $n \geq 0$, define $L_n = \{1^x01^y01^z \mid x + y \equiv z \mod n\}$.

   Show how to construct a regular expression that accepts L_n.

3. Recall the language L_k from the homework, which was defined to be the set of all binary strings with length at least k that have a 1 in the kth-to-last position.

   (a) Draw a state diagram for a 4-state NFA with L_3 as its language.

   (b) Use the subset construction to create a DFA with the same language.

   (c) Is the DFA you provided the minimal DFA for L_3? Explain.

4. Let L be the language of balanced parentheses with alphabet $\Sigma = \{\texttt{(},\texttt{)}\}$. For example, `(())()` $\in L$ but `((())()` $\notin L$.

   Prove that L is not regular.

5. For each of the following languages, say whether L_i is decidable and whether L_i is recognizable, and give a short proof of your claim. (If you prove decidability, the proof of recognizability is not required.)

   (a) $L_1 = \{M \mid \text{the Turing machine } M \text{ has 154 states}\}$

   (b) $L_2 = \{(M, w) \mid \text{the DFA } M \text{ rejects input } w\}$

   (c) $L_3 = \{M \mid \text{TM } M \text{ accepts some string of length greater than 154}\}$

   (d) $L_4 = \{M \mid M \text{ is a TM and } L(M) \text{ is not regular}\}$

6. Suppose for two languages A and B, that $\bar{A} \leq_M \bar{B}$ (i.e. the complement of A is mapping reducible to the complement of B). Which of the following are necessarily true?

   (a) If B is empty, then A is empty.

   (b) If B is regular, then A is regular.

   (c) If A is decidable, then B is decidable.

   (d) If A is undecidable, then B is undecidable.

   (e) If B is recognizable, then A is recognizable.