Practice Midterm

Will not be graded.

Problems

1. Let \( \Sigma = \{0, 1\} \) and for all \( n \geq 0 \), define \( L_n = (\{0, 1\}^*0\{0, 1\}^n0) \cup (\{0, 1\}^*1\{0, 1\}^n1) \).
   Show how to construct a DFA that accepts \( L_n \).

2. Let \( \Sigma = \{1, 0\} \) and for all \( n \geq 0 \), define \( L_n = \{1^x01^n1^y \mid x + y \equiv z \mod n \} \).
   Show how to construct a regular expression that accepts \( L_n \).

3. Recall the language \( L_k \) from the homework, which was defined to be the set of all binary strings with length at least \( k \) that have a 1 in the \( k \)th-to-last position.
   (a) Draw a state diagram for a 4-state NFA with \( L_3 \) as its language.
   (b) Use the subset construction to create a DFA with the same language.
   (c) Is the DFA you provided the minimal DFA for \( L_3 \)? Explain.

4. Let \( L \) be the language of balanced parentheses with alphabet \( \Sigma = \{\),\( \} \). For example, \( ()() \in L \) but \( ((())()) \notin L \).
   Prove that \( L \) is not regular.

5. For each of the following languages, say whether \( L_i \) is decidable and whether \( L_i \) is recognizable, and give a short proof of your claim. (If you prove decidability, the proof of recognizability is not required.)
   (a) \( L_1 = \{M \mid \text{the Turing machine } M \text{ has 154 states}\} \)
   (b) \( L_2 = \{(M, w) \mid \text{the DFA } M \text{ rejects input } w\} \)
   (c) \( L_3 = \{M \mid \text{TM } M \text{ accepts some string of length greater than 154}\} \)
   (d) \( L_4 = \{M \mid M \text{ is a TM and } L(M) \text{ is not regular}\} \)
6. Suppose for two languages $A$ and $B$, that $\bar{A} \leq_M \bar{B}$ (i.e. the complement of $A$ is mapping reducible to the complement of $B$). Which of the following are necessarily true?

(a) If $B$ is empty, then $A$ is empty.
(b) If $B$ is regular, then $A$ is regular.
(c) If $A$ is decidable, then $B$ is decidable.
(d) If $A$ is undecidable, then $B$ is undecidable.
(e) If $B$ is recognizable, then $A$ is recognizable.