Practice Midterm 1

1. State whether each of the following statements is true. In addition, give a short proof (2-3 lines are sufficient) if the statement is true, and give a counterexample otherwise.

   (a) Fix $\Sigma = \{0, 1\}$. If L is regular, then the following language must be regular:

   $$\{w \mid w \in L \text{ and } w \text{ ends in } 10101\ \}$$

   (b) There are infinitely many Turing-recognizable languages.

   (c) If two languages L_1 and L_2 over the same alphabet Σ are decidable, then $L_1 \triangle L_2$ is decidable. (Note: the symmetric difference $S \triangle T$ of two sets S, T is defined to be the set of elements belonging to S or T but not both.)

2. (Sipser 1.37) Consider the language

   $$F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

   over alphabet $\Sigma = \{a, b, c\}$.

   (a) Show that F is not regular.

   (b) Show that F satisfies the 3 conditions of the pumping lemma. (That is, for every pumping length $p \geq 1$, and for every string $w \in F$ of length at least p, show that w can be written in a form that can be pumped and yet remain in F.)

   (c) Explain why (a), (b) do not contradict the pumping lemma.

3. For two languages L_1, L_2 over some alphabet Σ, define $\mathsf{shuffle}(L_1, L_2)$ to be the set of strings that can be formed by interleaving a string from L_1 with a string from L_2. In this interleaving, the symbols from the two strings need not alternate at every step, but their order must be the same as in the two original strings. (So, for example, some valid interleavings of the strings 'mickey' and 'mouse' are 'mickeymouse', 'mousemickey' and 'mimckeousey'.) Show that if L_1 and L_2 are regular, then $\mathsf{shuffle}(L_1, L_2)$ is regular.

4. (Sipser 1.31) Consider a new kind of finite automaton called a coNFA (also referred to as all-paths-NFA in Sipser 1.31). A coNFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if *every* possible computation of M on x ends in a state from F. Note, in contrast, that an ordinary NFA accepts a string if *some* computation ends in an accept state.

   (a) Prove that the class of languages recognized by coNFAs is the class of regular languages.

   (b) Give a polynomial time algorithm that on input a coNFA M and a string $x \in \Sigma^*$, decides if M accepts x.

5. Consider the language

   $$A_{\mathsf{DFAinf}} = \{D \mid D \text{ is a DFA that recognizes a language containing infinitely many strings}\}$$

   Prove that A_{DFAinf} is decidable.

6. (Sipser 4.17) Let C be a language. Prove that C is Turing-recognizable (recursively enumerable) iff a decidable language D exists such that $C = \{x \mid \exists\, y \text{ such that } \langle x, y\rangle \in D\}$.