Practice Midterm 2

1. Consider the language

$$INT_{\mathsf{TM}} = \{\langle M_1, M_2 \rangle : L(M_1) \cap L(M_2) \neq \emptyset\}.$$

(Thus, INT_{TM} is the language associated with the problem of deciding whether, for two given Turing machines M_1 and M_2, there is some string that is accepted by both machines.)

   (a) Show that INT_{TM} is Turing recognizable.

   (b) Show that INT_{TM} is not decidable.

   (c) Is the language

$$EINT_{\mathsf{TM}} = \{\langle M_1, M_2 \rangle : L(M_1) \cap L(M_2) = \emptyset\}.$$

   Turing recognizable?

2. Consider the following time-bounded variant of Kolmogorov complexity, written $K_L(x)$, and defined to be the shortest string $\langle M, w, t \rangle$ where t is a positive integer written in binary, and M is a TM that on input w halts with x on its tape without t steps.

   (a) Show that $K_L(x)$ is computable (by describing an algorithm that on input x outputs $K_L(x)$).

   (b) Prove that for all positive integers n, there exists a string x of length n such that $K(x) = O(\log n)$ and $K_L(x) \geq n$. (In fact, there is an algorithm that on input n finds such a x.)

3. (Sipser 7.36) For a cnf-formula ϕ with m variables and c clauses (that is, ϕ is the AND of c clauses, each of which is an OR of several variables), show that you can construct in polynomial time an NFA with $O(cm)$ states that accepts all nonsatisfying assignments, represented as Boolean strings of length m. Conclude that the problem of minimizing NFAs (that is, on input a NFA, find the NFA with the smallest number of states that recognizes the same language) cannot be done in polynomial time unless $\mathbf{P} = \mathbf{NP}$.

4. Prove that the halting problem $HALT_{\mathsf{TM}}$ for Turing machines is $\mathbf{NP}$-hard.