Practice Final

1. Sipser 1.42.
2. One of the following languages is Turing-recognizable; the other is not. Which is which?
   - $A = \{\langle M \rangle \mid M \text{ accepts at most 172 distinct inputs}\}$
   - $B = \{\langle M \rangle \mid M \text{ accepts more than 172 distinct inputs}\}$
3. Prove that the following language is **NP**-complete:

   $$\mathit{GraphEmbed} = \{\langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are undirected graphs and } G_2 \text{ can be embedded in } G_1\}$$

   (that is, G_2 is a subgraph of G_1).
4. Define

   $$\begin{aligned}\textsc{Cycle-Length} &= \{\langle G, c \rangle \mid 3 \leq c \leq |V(G)|, G \text{ is a directed graph and} \\ &\quad \text{the length of the shortest cycle in } G \text{ is } c.\}\end{aligned}$$

   Prove that $\textsc{Cycle-Length}$ is **NL**-complete.
5. Prove that $\mathbf{L} \neq \mathbf{TIME}(O(n))$.
6. In a one-message perfect zero-knowledge proof system, the prover only sends the verifier exactly one message, upon which the verifier must output accept or reject, without any further interaction (note that the verifier may still be a randomized algorithm). Prove that any language with a one-message perfect zero-knowledge proof system is in **BPP**.