

Problem Set 10

This problem set is due on **Friday Apr 23, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. Define the language

$$\text{SHORTESTPATH} = \{(G, k, s, t) \mid \text{the shortest path from } s \text{ to } t \text{ in } G \text{ has length } k\}$$

- (a) Prove that SHORTESTPATH is in **NL**.
- (b) Prove that SHORTESTPATH is in **L** if and only if **L** = **NL**.

2. Define the language

$$\#SAT = \{(\varphi, k) \mid \varphi \text{ is a 3CNF that has precisely } k \text{ satisfying assignments}\}$$

Prove that if $\#SAT \in \mathbf{NP}$ then $\mathbf{NP} = \mathbf{coNP}$.

3. Prove that E_{DFA} is **NL**-complete.
4. Define $EQ_{\text{NFA}} = \{\langle N_1, N_2 \rangle \mid N_1, N_2 \text{ are NFAs and } L(N_1) = L(N_2)\}$. Prove that $EQ_{\text{NFA}} \in \mathbf{PSPACE}$.