Problem Set 11

This problem set is due on **Friday Apr 30, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

(This version was posted on 4/26/04.)

1. Define

$$A_{\mathsf{LBA}} = \{\langle M, w\rangle \mid \quad M \text{ is a linear bounded automaton (LBA) with tape alphabet of size four that accepts } w\}$$

(Refer to Sipser page 177 for the definition of a LBA; it is essentially a Turing machine that uses linear space.)

   (a) Prove that A_{LBA} is **PSPACE**-complete under logarithmic space reductions, and deduce that A_{LBA} is also **PSPACE**-complete under polynomial-time reductions.

   (b) Prove that $A_{\mathsf{LBA}} \notin \mathbf{NL}$.

   (c) Prove that $A_{\mathsf{LBA}} \in \mathbf{SPACE}(O(n))$.

   (d) Prove that $\mathbf{P} \neq \mathbf{SPACE}(O(n))$.
   [Note: do not try to prove $A_{\mathsf{LBA}} \notin \mathbf{P}$, which is probably true but hopeless. Instead, assume $\mathbf{P} = \mathbf{SPACE}(O(n))$ and then, using the previous results and other things you know about **P**, deduce a contradiction.]

2. Sipser 9.18. Correct the last two lines as in the Errata, so that it reads

$$pad(A, f(n)) = \{pad(s, f(n)) \mid s \in A, \text{ where } n \text{ is the length of } s\ \} .$$

   Prove that if $A \in \mathbf{TIME}(O(n^6))$ then $pad(A, n^2) \in \mathbf{TIME}(O(n^3))$.

3. Sipser 9.19. Define $\mathbf{EXPTIME} = \mathbf{TIME}(2^{n^{O(1)}})$ and $\mathbf{NEXPTIME} = \mathbf{NTIME}(2^{n^{O(1)}})$. Show that $\mathbf{EXPTIME} \neq \mathbf{NEXPTIME}$ implies $\mathbf{P} \neq \mathbf{NP}$.