

## Problem Set 12

This problem set is due on **Tuesday May 11, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage [www.cs.berkeley.edu/~luca/cs172](http://www.cs.berkeley.edu/~luca/cs172).

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1. Prove a hierarchy theorem for circuit size. For concreteness, show that there is a constant  $c$  such that for every sufficiently large  $n$  there is a Boolean function  $f : \{0, 1\}^n \rightarrow \{0, 1\}$  such that  $f \notin \mathbf{SIZE}(n^3)$ , but  $f \in \mathbf{SIZE}(cn^3 \log n)$ .

[Note: you might be able to show  $f \in \mathbf{SIZE}(cn^3)$ , or even  $f \in \mathbf{SIZE}(n^3 + cn)$ . There is a construction that achieves  $f \in \mathbf{SIZE}(n^3 + c)$ .]

2. Show that there is a language  $L$  that can be solved by a family of read-once branching programs of size  $O(n)$  but such that any automaton that decides  $L \cap \{0, 1\}^n$  must have  $2^{\Omega(n)}$  states.
3. Sipser Problem 10.11.
4. Sipser Problem 10.19.