Problem Set 12

This problem set is due on **Tuesday May 11, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. Prove a hierarchy theorem for circuit size. For concreteness, show that there is a constant c such that for every sufficiently large n there is a Boolean function $f : \{0,1\}^n \to \{0,1\}$ such that $f \notin \mathbf{SIZE}(n^3)$, but $f \in \mathbf{SIZE}(cn^3 \log n)$.

   [Note: you might be able to show $f \in \mathbf{SIZE}(cn^3)$, or even $f \in \mathbf{SIZE}(n^3 + cn)$. There is a construction that achieves $f \in \mathbf{SIZE}(n^3 + c)$.]

2. Show that there is a language L that can be solved by a family of read-once branching programs of size $O(n)$ but such that any automaton that decides $L \cap \{0,1\}^n$ must have $2^{\Omega(n)}$ states.

3. Sipser Problem 10.11.

4. Sipser Problem 10.19.