Problem Set 2

This problem set is due on **Friday February 6, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. Sipser problem 1.41.
2. A string is a palindrome if it reads the same way forward and backward, like `radar`, or `1101011`. Show that for every alphabet Σ (with $|\Sigma| \geq 2$), the language of palindromes over Σ is not regular.
3. (a) Let A be the set of strings over $\{0,1\}$ that can be written in the form $1^k y$ where y contains at least k 1s, for some $k \geq 1$. Show that A is a regular language.

   [Note that the same string could fit the definition for more than one value of k. For example 1101010 can be seen as 1 followed by the string $y = 101010$, which contains at least one 1, or as 11 followed by 01010. On the other hand, the string 100, for example, is not in A because there is no value of k for which the definition applies.]

   (b) Let B be the set of strings over $\{0,1\}$ that can be written in the form $1^k 0 y$ where y contains at least k 1s, for some $k \geq 1$. Show that B is not a regular language.

   (c) Let C be the set of strings over $\{0,1\}$ that can be written in the form $1^k z$ where z contains at most k 1s, for some $k \geq 1$. Show that C is not a regular language.
4. Let k be a positive integer. Let $\Sigma = \{0,1\}$, and L be the language consisting of all strings over $\{0,1\}$ containing a 1 in the kth position from the end (in particular, all strings of length less than k are not in L).

   (a) Prove that any DFA that recognizes L has at least 2^k states.

   (b) Prove that any NFA that recognises L has at least k states.