Problem Set 5

This problem set is due on **Friday March 5, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

1. Sipser problem 4.19.
2. Sipser problem 5.14.
3. (a) Prove that $\overline{E_{\mathsf{TM}}}$ is Turing-recognizable.
   (b) Prove that A_{TM} is not mapping reducible to E_{TM}.
4. For each of the following languages, give a proof that it is undecidable or describe an algorithm to decide it. (You may assume that all the languages are over the alphabet $\{0,1\}$ and all the Turing machines have $\{0,1\}$ as their input alphabet.)
   (a) $L_1 = \{\langle M \rangle \mid M$ is a Turing machine that rejects all inputs of even length$\}$.
   (b) $L_2 = \{\langle M \rangle \mid M$ is a Turing machine that halts on an empty input$\}$.
   (c) $L_3 = \{\langle M \rangle \mid$ there is some input $x \in \{0,1\}^*$ such that M accepts x in less than 100 steps $\}$.