Problem Set 1

This problem set is due on **Thursday September 8th, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. Prove regularity two different ways by doing both of the following:

   (i) Give a regular expression representing the language

   (ii) Exhibit DFA/NFA that recognizes the language

   for the following languages:

   (a) The set of string for which the number of a's is divisible by five.

   (b) The set of strings which contain the substring abb at least twice.

2. We say that $w = w_1 w_2 \dots w_n$ is a *final segment* of $v = v_1 v_2 \dots v_k$ if $n \leq k$ and for every $i \in \{1, 2, \dots, n\}$, $w_i = v_{i+k-n}$. In other words, w is some last part of v. For any language A, define:

   $$A^{final} = \{y \mid y \text{ is a final segment of } x \text{ for some } x \in A\}$$

   Show that if A is regular, then so is A^{final}.

3. Fix an alphabet Σ. Let f be any function from Σ into Σ^*. In other words f assigns a string to each letter of the alphabet. For any string $w = w_1 w_2 \dots w_n$, let

   $$f[w] = f(w_1) \circ f(w_2) \circ \dots \circ f(w_n)$$

   I.e. $f[w]$ is made by replacing each letter of w with the string assigned by f. For any language A, define:

   $$^f A = \{y \mid y = f[x] \text{ for some } x \in A\}$$

   Show that if A is a regular language, then so is $^f A$.