Problem Set 10

This problem set is due on **Monday, November 28 by 5:00pm.** Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. For a graph $G = (V, E)$, a *dominating set* is a collection of vertices $X \subset V$ such that for any vertex v, either $v \in X$ or there is an edge $\{v, u\} \in E$ such that $u \in X$. Define the language

   $$DS = \{(G, k) \mid G \text{ is a graph with a dominating set of size at most } k\}$$

   Prove that DS is **NP**-complete.

   [*Hint:* reduce from vertex cover.]

2. In the *Integer Linear Programming* problem (abbreviated ILP), the input is a a set of inequalities such as:

   $$3x_1 + 2x_2 - 2x_4 \leq 7$$
   $$2x_1 - x_4 + x_7 - 3x_9 \geq -2$$
   $$\vdots$$

   That is, each inequality has the form:

   $$a_1x_1 + a_2x_2 + \ldots a_nx_n \leq c$$

   where c and the a_i are integers.

   The ILP problem is: given such a system of inequalities, is there a way to assign integer values to the variables x_i so that all the inequalities are true? Show that Integer programming is **NP**-hard.

   [*Hint:* reduce from $3SAT$, and force all the variables to be 0 or 1.]

3. Recall that the language $CLIQUE = \{(G, k) \mid G \text{ is a graph with a clique of size } k\}$ is **NP**-complete. Now we consider the language:
   $MAXCLIQUE = \{(G, k) \mid G \text{ is a graph whose largest clique is size exactly } k\}$

   (a) Show that $MAXCLIQUE$ is **NP**-hard.

   (b) Show that $MAXCLIQUE$ is *co***NP**-hard. (I.e. show that $\overline{MAXCLIQUE}$ is **NP**-hard.)

   (c) What would happen if $MAXCLIQUE$ were in **NP**?