Problem Set 5

This problem set is due on **Thursday October 6th, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. (From Sipser's book.) Fix the alphabet $\Sigma = \{0, 1\}$. Show that a language L over Σ is decidable if and only if there is an enumerator for L that outputs the strings of L in lexicographic order.

2. Say that string x is a *prefix* of string y if a string z exists where $xz = y$, and say that x is a *proper prefix* of y if in addition $x \neq y$. A language is *prefix-free* if it doesn't contain a proper prefix of any of its members. For example, $(01 \cup 00)^*1$ is prefix-free, but $(0 \cup 1)^*10$ is not.

   Let

   $$\mathsf{PrefixFree}_{\mathsf{REX}} = \{R | R \text{ is a regular expression over } \{0,1\} \text{ where } L(R) \text{ is prefix-free}\}$$

   (Such a language is defined over the alphabet $\{0, 1, \cup, (,), ^*, \circ\}$.) Show that $\mathsf{PrefixFree}_{\mathsf{REX}}$ is decidable.

3. Let $\Sigma = \{l_1, l_2, \ldots, l_k\}$ be some finite alphabet. Let L be some Turing-recognizable language with alphabet Σ. Suppose we pick a way to code Σ with binary strings of length n for some natural number n. I .e. we pick a function $f : \Sigma \to \{0,1\}^n$ so that no two letters have the same code. Show that there is a Turing machine M with the following properties:

   (i) The input alphabet for M is $\{0, 1\}$

   (ii) The tape alphabet for M is $\{0, 1, \sqcup\}$.

   (iii) M rejects all inputs which are not codes of strings in Σ^*.

   (iv) M accepts all strings which are codes of strings in L.

   Hint: Simulate a Turing machine for L. The machine M should be able to "remember" an entire fixed length block of letters.