Problem Set 6

This problem set is due on **Thursday October 20th, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

Typo in Problem 2 corrected on 10/17

1. Let $F = \{\langle M \rangle \mid M$ halts on every input of length less than 2^{1000} in less than 2^{1000} steps $\}$. Show that F is decidable.

2. Let $B = \{(n, m) \mid$ Every n- state machine M either halts in less than m steps on an empty input, or doesn't halt on an empty input$\}$.

   (a) Show that B is not decidable.

   (b) Show that B is not recognizable.
   Hint: Show that $\overline{B}$ is recognizable.

3. Let $H_{TM}^{1/2} = \{(\langle M \rangle, x, y) \mid M$ halts on x but not on $y\}$.

   (a) Show that $\overline{H_{TM}} \leq_m H_{TM}^{1/2}$.

   (b) Use part (a) to show that neither $H_{TM}^{1/2}$ nor $\overline{H_{TM}^{1/2}}$ is recognizable.