Problem Set 7

This problem set is due on **Thursday October 27th, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. (Sipser 5.9) Let $T = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable. Use Rice's theorem: show that $T = L_{\mathcal{C}}$, for a properly defined $\mathcal{C}$, and show that T is non-empty and does not contain all Turing machines.

   [Recall that, for a string $w = (w_1, \ldots, w_n)$, we define $w^R = (w_n, \ldots, w_1)$.]

2. Suppose B is an undecidable language such that $B \leq_m \overline{B}$. Prove that neither B nor $\overline{B}$ is Turing-recognizable.

3. Show that if A is recognizable, then there is some decidable language D so that:

$$x \in A \Leftrightarrow \exists y. \langle x, y \rangle \in D$$