CS 172 Spring 2007 — Discussion Handout 1

1. **Simpleton machines: DFAs**
   Design DFAs to recognize the following languages:

   (a) $\{w \mid w$ is any string not in $a^*b^*\}$ with $\Sigma = \{a, b\}$.

   (b) $\{w \mid w$ has length at least 3 and its third symbol is $0\}$ with $\Sigma = \{0, 1\}$.

   (c) $\{w \mid w$ contains an even number of a's and an odd number of b's and does not contain the substring ab$\}$, $\Sigma = \{a, b\}$.

   (d) $B_n = \{a^k \mid n$ divides $k\}$ for $\Sigma = \{a\}$.

2. **Getting Moody: NFAs**
   Design NFAs to recognize the following languages:

   (a) The set of all binary strings (of length at least 10) such that at least one of the last 10 characters is a 1.

   (b) The set of all decimal numbers such that the final digit has not appeared before.

3. **Once a regular language, always a regular language**
   In the lecture you saw certain operations like union, intersection, star etc., which when applied to a regular language (or two languages), still give a regular language. Here we define some more operations on a single language. Prove that if A is a regular language, then $Op(A)$ is also a regular language, for each of the operations defined below.

   (a) **Complement:** $A^c = \{w \in \Sigma^* \mid w \notin A\}$.

   (b) **NOPREFIX:** $NOPREFIX(A) = \{w \in A \mid$ No proper prefix of w is in $A\}$.

   (c) **DROP-OUT:** $DROP-OUT(A) = \{xz \mid x, z \in \Sigma^*$ and $\exists y \in \Sigma$ such that $xyz \in A\}$.

4. **Laconic NFAs**
   Show that every NFA can be converted to another NFA which accepts exactly the same language, but has just one accepting state.