

## CS 172 Spring 2007 — Discussion Handout 12

### 1. Memories of the past

We say that a directed graph is strongly connected if every pair of nodes is connected by a directed path in each direction. Let

$$\text{STRONGLY-CONNECTED} = \{\langle G \rangle \mid G \text{ is a strongly connected graph}\}$$

Show that STRONGLY-CONNECTED is NL-complete.

### 2. Its hard to be concise

We say that two boolean formulas are equivalent if they have the same set of variables and are true on the same assignments. A formula is said to be minimal if no shorter formula is equivalent to it. Let

$$\text{MIN} - \text{FORMULA} = \{\varphi \mid \varphi \text{ is a minimal boolean formula}\}$$

(a) Show that  $\text{MIN} - \text{FORMULA} \in \text{PSPACE}$ .

(b) Explain why the following argument fails to show that  $\text{MIN} - \text{FORMULA} \in \text{coNP}$ :

If  $\varphi \notin \text{MIN} - \text{FORMULA}$ , then  $\varphi$  has a smaller equivalent formula. An NTM can verify that  $\varphi \in \overline{\text{MIN} - \text{FORMULA}}$  by guessing that formula.

### 3. A (NL) hard problem on DFAs!

Prove that  $E_{\text{DFA}}$  is NL-complete.

*Hint:* You may use the fact that  $\text{coNL} = \text{NL}$ .