CS 172 Spring 2007 — Discussion Handout 3

1. **Some more irregularity...**
   Prove that the following languages are not regular. Can you also give an infinite set of pairwise distinguishable strings to directly show that the number of equivalence classes in the indistinguishability relation must be infinite.

   (a) $\{0^i 1^j \mid \gcd(i, j) = 1\}$.

   (b) $\{x_1 \# x_2 \# \ldots \# x_k \mid k \geq 0, x_i \in 1^*, x_i \neq x_j \text{ for } i \neq j\}$.

2. **Much ado about nothing?**
   Give a polynomial time algorithm which, given a DFA for a regular language, decides if the language is empty (does not contain any strings).

3. **Short witnesses for large languages**
   Show that for any deterministic finite automaton M, M accepts an infinite language iff M accepts some string of length ℓ, where $|Q| \leq \ell < 2|Q|$. Here, Q is the set of states of M.
   Hint: Use the pumping lemma.

4. **Half a language**
   [**Caution: Tricky problem ahead!**]
   If A is any language, let $A_{\frac{1}{2}-}$ be the set of first halves of strings in A so that

   $$A_{\frac{1}{2}-} = \{x \mid \text{for some } y, |x| = |y| \text{ and } xy \in A\}$$

   Show that if A is regular, then so is $A_{\frac{1}{2}-}$.
   Hint: You might need to guess the second half as you are reading the first half and keep track of the state the machine "would" reach had it read the second half you guessed.