CS 172 Spring 2007 — Discussion Handout 4

1. **The multi-headed monster**
   We define a *Hydra Machine* as a Turing machine with k-heads (numbered $1 \ldots k$) instead of one, all of which read and write the same tape. All heads start in the same cell at the beginning of the input and at each step, the machine can read all the cells where its heads are, change its state and move each head left or right. Show that a hydra machine is no more powerful than a Turing machine.

2. **Zero or One?**
   Let L be a Turing-recognizable language and let $\bar{L}$ be non-Turing recognizable. Define

   $$L' = \{0w \mid w \in L\} \cup \{1w \mid w \notin L\}$$

   What can you say about L' ? Is it decidable or Turing recognizable or neither? What about $\bar{L}'$?

3. **The hidden decidable**
   Show that any infinite, Turing-recognizable (recursively enumerable) language contains an infinite, decidable language.
   Hint: Consider the enumerator for the language. Can you select a few strings from its output (to make the subset) in such a manner that after some time you can be sure whether a given string will appear in this selection or not?