CS 172 Spring 2007 — Discussion Handout 5

1. **To decide or not to decide**
   For each of the following languages, give a proof that it is undecidable or describe an algorithm to decide it. (You may assume that all the languages are over the alphabet $\{0, 1\}$ and all the Turing machines have $\{0, 1\}$ as their input alphabet.)

   (a) $L_1 = \{\langle M \rangle \mid M$ is a Turing machine that rejects all inputs of even length$\}$.

   (b) $L_2 = \{\langle M \rangle \mid M$ is a Turing machine that halts on an empty input$\}$.

   (c) $L_3 = \{\langle M \rangle \mid$ there is some input $x \in \{0, 1\}^*$ such that M accepts x in less than 100 steps $\}$.

2. **More on halting**
   $H_{TM}^{1/2} = \{(\langle M \rangle, x, y) \mid M$ halts on x but not on $y\}$.

   (a) Show that $\overline{H_{TM}} \leq_m H_{TM}^{1/2}$.

   (b) Use part (a) to show that neither $H_{TM}^{1/2}$ nor $\overline{H_{TM}^{1/2}}$ is recognizable.

3. **Erasers are hard to find**
   Consider the problem of testing whether a given single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any string input to it i.e. does it ever erase anything on the tape. Formulate this problem as a language, and show that it is undecidable.

4. **Consent in its refusal**
   Let L be a Turing-recognizable language and let $\bar{L}$ be non-Turing recognizable. We defined the following language in the previous discussion:

$$L' = \{0w \mid w \in L\} \cup \{1w \mid w \notin L\}$$

   (a) Show that $L' \leq_m \overline{L'}$.

   (b) Show that for any undecidable language having the property that $B \leq_m \overline{B}$, neither B nor $\overline{B}$ is Turing-recognizable.