CS 172 Spring 2007 — Discussion Handout 6

1. **The hardest of them all**
   Show that a language A is:

   (a) Turing-recognizable iff $A \leq_m A_{TM}$.

   (b) decidable iff $A \leq_m 0^*1^*$.

2. **More on Rice's theorem**
   In Rice's theorem, we prove that a language L consisting of Turing machine descriptions such that the language of the TMs belong to a class $\mathcal{C}$, is undecidable. We assumed two properties of L:

   (a) L is nontrivial i.e. L is not empty or equal to the set of all Turing machines.

   (b) If $L(M_1) = L(M_2)$, then $\langle M_1 \rangle \in L \Leftrightarrow \langle M_2 \rangle \in L$.

   Prove that both these properties are necessary for proving L to be undecidable.

3. **When recognizability met decidability**
   Let C be a language. Prove that C is Turing-recognizable iff a decidable lannguage D exists such that $C = \{x \mid \exists y(\langle x, y \rangle \in D)\}$.
   Hint: Think of y as a proof that $x \in C$. What can be a good proof?

4. **Cantor's ghost**
   Let S be a set and let $P(S)$ be the set of all the subsets of S. Show that $|P(S)| > |S|$.
   Hint: First show this for the set of natural numbers.