Problem Set 1

This problem set is due on **Wednesday January 31, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. Prove that the following languages are regular, either by exhibiting a regular expression representing the language, or a DFA/NFA that recognizes the language:

   (a) all strings that do not contain the substring aba, for $\Sigma = \{a, b\}$ (for instance, $aabaa$ contains the substring aba, whereas $abba$ does not)

   (b) set of strings such that each block of 4 consecutive symbols contains at least two a's, for $\Sigma = \{a, b\}$

   (c) set of binary strings ($\Sigma = \{0, 1\}$) which when interpreted as a number (with the most significant bit on the left), are divisible by 5.

2. (Sipser, problem 1.31) For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written as w^R is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R \mid w \in A\}$. Show that if A is regular, so is A^R.

3. We say a string $w = w_1 w_2 \ldots w_n$ is a *shuffle* of strings u and v if there exists $J \subseteq \{1, \ldots, n\}$ such that $(w_j)_{j \in J} = u$ and $(w_j)_{j \notin J} = v$. For example CSS17PR2ING07 is a shuffle of the strings CS172 and SPRING07 and in fact, there are two sets $J = \{1, 2, 4, 5, 8\}$ and $J = \{1, 3, 4, 5, 8\}$ which work here.

   We then define the shuffle of two languages A and B as

   $$S(A, B) = \{w | \exists\ u \in A, v \in B \text{ s.t. } w \text{ is a shuffle of } u \text{ and } v\}$$

   Show that if A and B are regular languages over a common alphabet Σ, then so is $S(A, B)$.