Problem Set 2

This problem set is due on **Wednesday February 7, by 4:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

---

1. Let k be a positive integer. Let $\Sigma = \{0, 1\}$, and L be the language consisting of all strings over $\{0, 1\}$ containing a 1 in the kth position from the end (in particular, all strings of length less than k are not in L).
   (a) Construct a DFA with exactly 2^k states that recognizes L.
   (b) Construct a NFA with exactly $k + 1$ states that recognizes L.
   (c) Prove that any DFA that recognizes L has at least 2^k states.

2. (a) Let A be the set of strings over $\{0, 1\}$ that can be written in the form $1^k y$ where y contains at least k 1s, for some $k \geq 1$. Show that A is a regular language.
   [Note that the same string could fit the definition for more than one value of k. For example 1101010 can be seen as 1 followed by the string $y = 101010$, which contains at least one 1, or as 11 followed by 01010. On the other hand, the string 100, for example, is not in A because there is no value of k for which the definition applies.]
   (b) Let B be the set of strings over $\{0, 1\}$ that can be written in the form $1^k 0y$ where y contains at least k 1s, for some $k \geq 1$. Show that B is not a regular language.
   (c) Let C be the set of strings over $\{0, 1\}$ that can be written in the form $1^k z$ where z contains at most k 1s, for some $k \geq 1$. Show that C is not a regular language.

3. Write regular expressions for the following languages:
   (a) The set of all binary strings such that every pair of adjacent 0's appears before any pair of adjacent 1's.
   (b) The set of all binary strings such that the number of 0's in the string is divisible by 5.

4. We say a string x is a *proper prefix* of a string y, if there exists a non-empty string z such that $xz = y$. For a language A, we define the following operation

$$NOEXTEND(A) = \{w \in A \mid w \text{ is not a proper prefix of any string in } A\}$$

   Show that if A is regular, then so is $NOEXTEND(A)$.