Problem Set 4

This problem set is due on **Wednesday February 21, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. (Sipser, Problem 3.13) A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form

   $$\delta : Q \times T \leftarrow Q \times T \times \{R, S\}$$

   At each point the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is *not* equivalent to the usual version. (*Hint:* Show that these machines only recognize regular languages).

2. (Sipser, Problem 3.18) Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

3. Say that string x is a *prefix* of string y if a string z exists where $xz = y$, and say that x is a *proper prefix* of y if in addition $x \neq y$. A language is *prefix-free* if it doesn't contain a proper prefix of any of its members. Let

   $$\mathsf{PrefixFree}_{\mathsf{REX}} = \{R | R \text{ is a regular expression where } L(R) \text{ is prefix-free}\}$$

   Show that $\mathsf{PrefixFree}_{\mathsf{REX}}$ is decidable.

4. Let $Non - Empty$ be the following language

   $$Non - Empty = \{< M > \mid M \text{ accepts some string}\}.$$

   Show that $Non - Empty$ is Turing recognizable.