
Problem Set 5

This problem set is due on **Wednesday March 14, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. Let $B = \{(n, m) \mid \text{Every } n\text{-state machine } M \text{ either halts in less than } m \text{ steps on an empty input, or doesn't halt on an empty input}\}$.
 - (a) Show that B is not decidable.
 - (b) Show that B is not recognizable.
2. (Sipser 5.9) Let $T = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable.
3. (Sipser problem 6.13.) Consider the theory $\text{Th}(\mathbb{Z}_5, +, \times)$ defined like the theory $\text{Th}(\mathbb{N}, +, \times)$ except that addition and multiplication are performed modulo 5.

We allow variables x_1, \dots, x_n, \dots , and

- for every three variables x_i, x_j, x_k , we have that $x_i + x_j = x_k \pmod{5}$ is an expression with free variables x_i, x_j, x_k and that $x_i \times x_j = x_k \pmod{5}$ is also an expression with free variables x_i, x_j, x_k ;
- If E_1, E_2 are expressions, having free variables X_1 and X_2 respectively, then $E_1 \vee E_2$ and $E_1 \wedge E_2$ are expressions, having free variables $X_1 \cup X_2$. We also have that $\neg E_1$ is an expression, with free variables X_1 .
- If E is an expression with free variables X , and $x_i \in X$, then $\exists x_i.E$ and $\forall x_i.E$ are expressions with free variables $X - \{x_i\}$.
- An expression with no free variables is a *statement*.

For example, the statement $\forall x.\exists y.(y + y = x \pmod{5})$ is true (try it), but the statement $\forall x.\exists y.(y \times y = x \pmod{5})$ is false (consider $x = 2$).

Show that $\text{Th}(\mathbb{Z}_5, +, \times)$ is decidable.