
Problem Set 6

This problem set is due on **Wednesday March 21, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. (Sipser problem 6.20.) Show how to compute the Kolmogorov complexity $K_U(x)$ of a string x with an oracle for A_{TM} .

The definition of an oracle is given in Sipser definition 6.20 on page 233. An oracle is essentially a subroutine. You could interpret this problem as asking for an algorithm that on input x , computes the descriptive complexity of x , that is, $K_U(x)$, using a subroutine for A_{TM} . On input $\langle M, w \rangle$ the subroutine will return 1 if M accepts w , and 0 otherwise. Whenever you invoke the subroutine on some input $\langle M, w \rangle$, use the terminology “query the A_{TM} oracle on input $\langle M, w \rangle$ ”.

For instance, the machine S in the proof that $HALT_{\text{TM}}$ is undecidable in Sipser theorem 5.1 (page 188-189) is an example of a algorithm for $HALT_{\text{TM}}$ using an oracle for A_{TM} . In that example, S only uses the A_{TM} subroutine once. In general (and for this problem), you are allowed to invoke the subroutine any number of times, and the oracle queries may be adaptive (that is, the next query may depend on the answers to the previous ones).

2. Prove that if R is the set of Kolmogorov random strings $\{x \in \Sigma^* | K(x) \geq |x|\}$ and A is a decidable subset of R , then A is finite.
3. If $f : \Sigma^* \rightarrow \mathbb{Z}_+$ is a computable function such that $f(x) \leq K(x) \forall x \in \Sigma^*$, then show that f must be bounded.
4. (Sipser problem 7.17.) Prove that if $\mathbf{P} = \mathbf{NP}$ then every language in \mathbf{P} , except \emptyset and Σ^* , is \mathbf{NP} -complete. Why can't \emptyset and Σ^* be \mathbf{NP} -complete?