
Problem Set 8

This problem set is due on **Thursday April 19, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. Define *SETCOVER* to be
 $\{(U, S_1, \dots, S_m, k) \mid \forall i, S_i \subset U, \text{ and there is } I \subseteq \{1, \dots, m\} \text{ with } |I| = k \text{ and } U = \bigcup_{i \in I} S_i\}$
Show that *SETCOVER* is **NP**-complete.

2. Define the language

$\text{SHORTESTPATH} = \{(G, k, s, t) \mid \text{the shortest path from } s \text{ to } t \text{ in } G \text{ has length } k\}$

- (a) Prove that *SHORTESTPATH* is in **NL**.
 - (b) Prove that *SHORTESTPATH* is in **L** if and only if **L** = **NL**.
3. (Sipser 8.9) A *ladder* is a sequence of strings s_1, s_2, \dots, s_k , wherein every string differs from the preceding one in exactly one character. For example the following is a ladder of English words, starting with “head” and ending with “free”: head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free.

Let $LADDER_{\text{DFA}} = \{(M, s, t) \mid M \text{ is a DFA and } L(M) \text{ contains a ladder of strings, starting with } s \text{ and ending with } t\}$. Show that $LADDER_{\text{DFA}}$ is in **PSPACE**.