

## Practice Midterm 1

1. State whether each of the following statements is true. In addition, give a short proof (2-3 lines are sufficient) if the statement is true, and give a counterexample otherwise.

- (a) If  $L_1, L_2, \dots, L_{172}$  are all regular languages, then the language  $\bigcap_{i=1}^{172} L_i$  is regular.  
(b) If  $L_1, L_2, L_3, \dots$  is an infinite sequence of regular languages, then the language  $\bigcap_{i=1}^{\infty} L_i$  is regular.

2. Let

$$L = \{\langle D \rangle, w \mid D \text{ is a DFA over the binary alphabet } \{0, 1\} \text{ that accepts } w\}$$

(Assume that the encoding of DFAs also uses the binary alphabet.)

- (a) Show that  $L$  is not regular.  
(b) Show that  $L$  is decidable.

3. Consider the language

$$INT_{\text{TM}} = \{\langle M_1, M_2 \rangle : L(M_1) \cap L(M_2) \neq \emptyset\}.$$

(Thus,  $INT_{\text{TM}}$  is the language associated with the problem of deciding whether, for two given Turing machines  $M_1$  and  $M_2$ , there is some string that is accepted by both machines.)

- (a) Show that  $INT_{\text{TM}}$  is Turing recognizable.  
(b) Show that  $INT_{\text{TM}}$  is not decidable.

4. Let  $S = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \{\langle M \rangle\}\}$ . Show that neither  $S$  nor  $\bar{S}$  is Turing-recognizable.