Practice Midterm 2

1. Consider the following time-bounded variant of Kolmogorov complexity, written $K_L(x)$, and defined to be the shortest string $\langle M, w, t \rangle$ where t is a positive integer written in binary, and M is a TM that on input w halts with x on its tape within t steps.

   (a) Show that $K_L(x)$ is computable (by describing an algorithm that on input x outputs $K_L(x)$).

   (b) Prove that for all positive integers n, there exists a string x of length n such that $K(x) = O(\log n)$ and $K_L(x) \geq n$. (In fact, there is an algorithm that on input n finds such a x.)

2. (Sipser 7.41) For a cnf-formula ϕ with m variables and c clauses (that is, ϕ is the AND of c clauses, each of which is an OR of several variables), show that you can construct in polynomial time an NFA with $O(cm)$ states that accepts all nonsatisfying assignments, represented as Boolean strings of length m. Conclude that the problem of minimizing NFAs (that is, on input a NFA, find the NFA with the smallest number of states that recognizes the same language) cannot be done in polynomial time unless $\mathbf{P} = \mathbf{NP}$.

3. (Sipser 7.33) Prove that the following language is NP-hard

$$D = \{\langle p \rangle \mid p \text{ is a polynomial in several variables having an integral root}\}$$

(The problem is in fact, undecidable. Turing first published the notion of a Turing machine and formalization of algorithms to prove the undecidability of this very problem.)