Problem Set 4

This problem set is due on **Monday, October 5, by 10am.**

Use the CS172 drop box.

Write your **name and your student ID number** on your solution. Write legibly. The description of your proofs should be as **clear** as possible (which does not mean **long** – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage [www.cs.berkeley.edu/~luca/cs172](http://www.cs.berkeley.edu/~luca/cs172).

There will be no class on Thursday, September 24. We will have a make-up lecture on Friday, October 2. We’ll decide the time of the October 2 lecture next Tuesday, looking for a time that minimizes conflicts. [http://ucfacultywalkout.com/](http://ucfacultywalkout.com/)

In problem 1, describe the Turing machine in some detail, but not to the point of actually specifying the states and drawing a diagram. The level of detail used in the book in Example 3.11 and Example 3.12 is ideal. You are free to use a multi-tape machine or any variant described in Chapter 4 if it makes your solution easier. In problems 2 and 3, describe the algorithms as pseudocode, as you would do in a CS170 assignment; the level of detail in the proof of Theorem 4.2 in the book is a good example.

1. [20 /100] Describe informally a Turing machine that decides the language

   \[ L := \{ p : p \text{ is a prime number} \} \]

2. [20 /100] Show that the Turing-recognizable languages are closed under the star operation. That is, prove that, if \( L \) is a recognizable language, then \( L^* \) is recognizable.

3. [25/100] Let \( E \) be an enumerator that enumerates a language \( L \) in lexicographic order. Show that \( L \) is decidable.

4. [35/100] A **read-only** Turing machine is a Turing machine that is not allowed to write on the tape. The transition function of a read-only Turing machine is of the form

   \[ \delta : Q \times \Sigma \cup \{\square\} \to Q \times \{L, R\} \]

   and the computation of a read-only Turing machine is defined in the expected way.

   Prove that if \( M \) is a read-only Turing machine then the language \( L(M) \) recognized by \( M \) is regular.

   [Hint: a possible approach is to show that if \( k \) is the number of states of \( M \) then the number of equivalence classes in \( \approx_{L(M)} \) is at most a constant that depends only on \( k \). Be careful that there are \( k \)-state read-once Turing machines that recognize languages with \( 2^{\Omega(k)} \) equivalence classes (can you prove it?), so if your approach leads, for example, to a \( O(k^2) \) state upper bound then something is wrong.]