Problem Set 6

This problem set is due on **Wednesday, October 21, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. [35] Consider the problem

$$Hxor := \{(\langle M\rangle, x, y) : M \text{ halts on precisely one of the inputs } x, y\}$$

Show that $Hxor$ is not decidable and that its complement is also not decidable.

2. [35] Here is a version of Rice's theorem that allows you to prove non-recognizability of languages. Say that a set $\mathcal{C}$ of languages is *monotone* if it is such that for every language A such that $A \in \mathcal{C}$ and for every language B such that $A \subseteq B$, we also have $B \in \mathcal{C}$. If $\mathcal{C}$ is a set of languages, define

$$L_{\mathcal{C}} := \{\langle M\rangle : M \text{ recognizes a language in } \mathcal{C}\}$$

Prove that if $\mathcal{C}$ is a set of languages such that

- $\mathcal{C}$ is *not* monotone,
- $L_{\mathcal{C}} \neq \emptyset$ and
- $L_{\mathcal{C}} \neq \{\langle M\rangle : M \text{ is a Turing machine }\}$

then $L_{\mathcal{C}}$ is not recognizable.

3. [30] Fix the input alphabet $\Sigma = \{0, 1\}$. Define the function $BB(n)$ to be maximum, over all Turing machines $M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R)$ that halt on an empty input and such that $|Q| \leq n$ and $|\Gamma| \leq n$, of the number of time steps that it takes for M to halt on an empty input. Note that, for every fixed n, $BB(n)$ is well defined and finite.

Define the language

$$B := \{(n, m) : BB(n) \leq m\}$$

Prove that B is not recognizable.