

DUE: Friday January 27th, 5:00pm

Problem 1

Let $\text{size}(f)$ be the size of the smallest circuit that encodes f . Let $T(n) \leq \frac{2^n}{10n}$.
Prove: There exists $f : \{0, 1\}^n \rightarrow \{0, 1\}$ such that $T(n) < \text{size}(f) \leq T(n) + 10n$.

Problem 2

$L \in \text{MajorityP}$ if and only if there exists a probabilistic polynomial time TM M_L such that

$$\Pr \left[M_L(x) \text{ accepts} \right] = \begin{cases} \leq 1/2 & \text{if } x \notin L \\ > 1/2 & \text{if } x \in L \end{cases}$$

Prove: If $\text{P} = \text{MajorityP}$, then $\text{P} = \text{NP}$.

Problem 3

Prove: There exists a decidable L such that $L \in \text{P/poly}$ and $L \notin \text{P}$.