Stanford University — CS 254: Computational Complexity | Homework 4
Luca Trevisan | February 1st, 2012

DUE: Friday February 10th, 5:00pm

Problem 1

A cycle of a directed graph $G = (V, E)$ is a sequence $(v_0, ..., v_{k-1})$ of vertices such that:
$\forall\ 0 \leq i < j < k$: $v_i \neq v_j$
$\forall\ 0 \leq i < k$: $(v_i, v_{(i+1)\%k}) \in E$

Let #Cycle(G) be the number of cycles in graph G.
Note: (v_0, v_1, v_2) *and* (v_1, v_2, v_0) *are considered two different cycles.*

Prove: If there is a deterministic polynomial time 2-approx to #Cycle, then P = NP.
Hint: You may use, without proof, that Directed Hamiltonian Cycle is NP-complete.

Problem 2

Prove: If P = NP, then Exp $\not\subset$ CircuitSize($2^n/10n$).

Problem 3

Let $\{\mathcal{D}_n\}$ be a family of distributions where each $\mathcal{D}_n$ is a distribution over $\{0, 1\}^n$.

We say that $\{\mathcal{D}_n\}$ is **polynomial time sampleable** if there exists a probabilistic polynomial time turing machine A such that:
$\forall n\ \forall x \in \mathcal{D}_n : \Pr[A(1^n) = x] = \mathcal{D}_n(x)$.

We say that $\{\mathcal{D}_n\}$ is **polynomial time computable** if there exists a deterministic polynomial time turing machine A such that:
$\forall n\ \forall x \in \mathcal{D}_n : A(x) = \sum\limits_{y<x} \Pr[\mathcal{D}_n(y)]$.

Prove: If all polynomial time sampleable distributions are polynomial time computable, then $\mathrm{P} = \mathrm{P}^{\#\mathrm{P}}$.