DUE: Friday March 2nd, 5:00pm

Problem 1

Def: $\mathrm{OR}_n : \{0,1\}^n \to \{0,1\}$ is the n-bit or function.
Let p be any polynomial such that
$\forall x \in \{0,1\}^n : p(x_1, ..., x_n) = \mathrm{OR}_n(x_1, ..., x_n)$.
Prove: $\deg(p) \geq n$

Problem 2

Def: $\mathrm{Majority} : \{0,1\}^* \to \{0,1\}$

$$\mathrm{Majority}(x) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i \geq n/2, \text{ where } n = |x| \\ 0 & \text{otherwise} \end{cases}$$

Prove: Majority can be computed by a family of log depth, poly size, bounded fan-in circuits.