CS 254 Final

Allowed: textbook, lecture notes. Forbidden: collaboration, search engine.

Grading

- Due 2014-03-18 @ 2pm.
- Please send all submissions (both LaTeXed and handwritten) to cs254-win1314-hw@lists.stanford.edu

Problem 1

Let $\Delta = \text{NP} \cap \text{coNP}$. Prove $\Delta = P^{\Delta}$, where $P^{\Delta} = \bigcup_{L \in \Delta} P^L$.

Problem 2

Prove that $\text{Space}(n) \neq \text{NP}$.

Problem 3

Suppose there exists constant q such that $\text{SAT} \in \text{PCP}_{1,2^{-q}}(\log(n), q)$. Prove $\text{P} = \text{NP}$.

Problem 4

Let $f : \{0,1\}^n \to \{0,1\}$ be the majority function, i.e. $f(x) = 1$ iff $\sum_i^n x_i > n/2$. Prove that f can be computed by a family of circuits of $O(\log n)$ depth, $\text{poly}(n)$ size, where the gates consists of not gates, 2-input or gates, and 2-input and gates.

Problem 5

Using Grover's algorithm, show that 3-coloring can be solved on a quantum computer in $O(2^{n/2} \cdot n^{O(1)})$ time, where n is the number of vertices.