Conclusions and Future Directions

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“Hardness in P” workshop, STOC 15’

<u>This talk:</u>

➡ Quick recap.

➡ What's next for Hardness in P?

Five of the many possible directions I am excited about.

Take a problem X in P, say in ***O(n²)*** time.

And prove that:

" X *probably cannot* be solved in ***$O(n^{2-e})$*** time. "

We imitate NP-hardness.

An $O(n^{1.9})$ algorithm for problem X —*reductions*→ A popular and plausible conjecture is falsified

Three popular conjectures

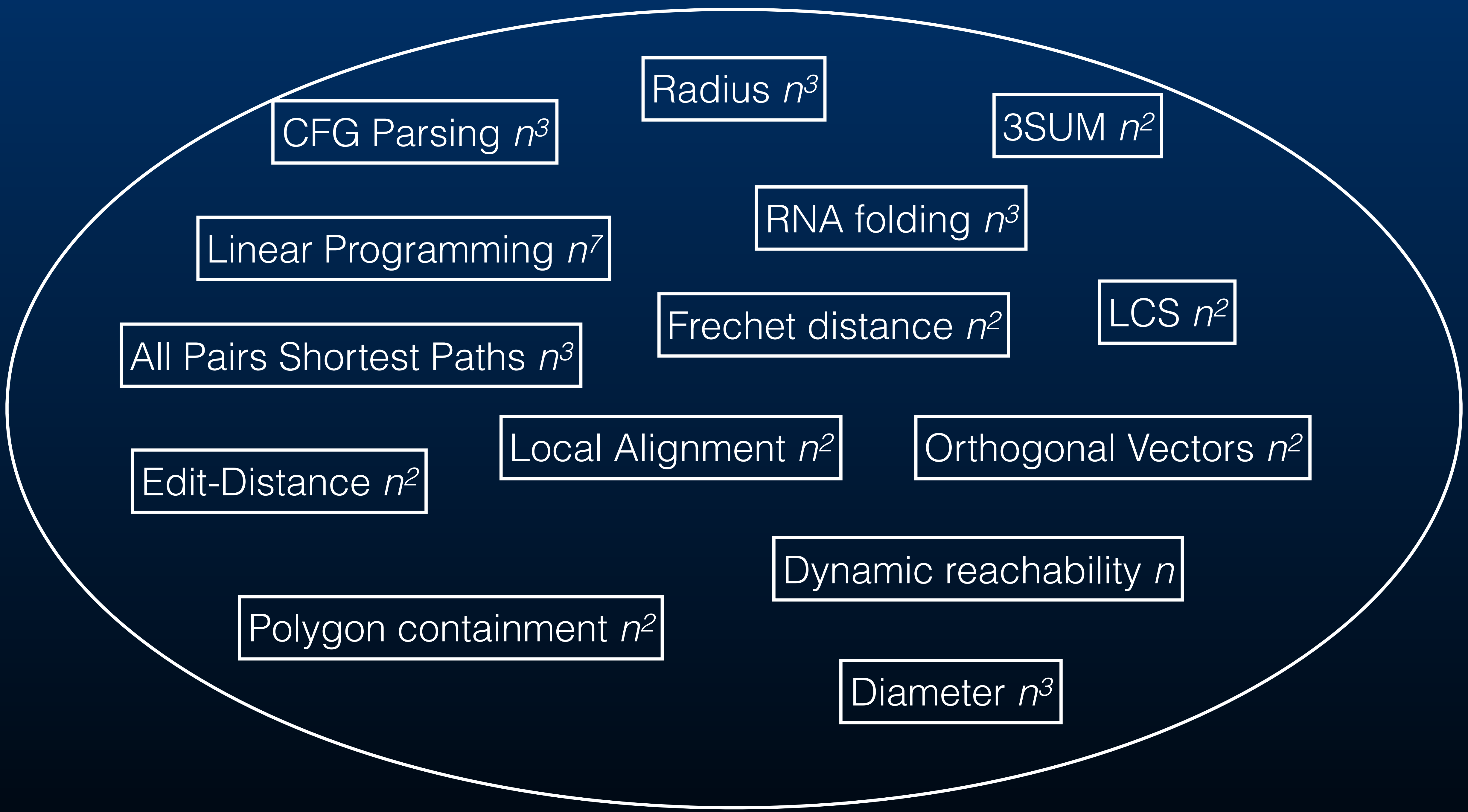
The 3-SUM Conjecture:
"No $O(n^{2-e})$ time algorithm for finding three numbers that sum to 0 in a list of n integers."

The APSP Conjecture:
"No $O(n^{3-e})$ time algorithm for computing all pairs shortest paths in an edge weighted graph on n nodes."

The Orthogonal Vectors Conjecture:
(implied by the Strong Exponential Time Hypothesis)
"No $O(n^{2-e})$ time algorithm for finding a pair of orthogonal vectors in a list of n boolean vectors of length $\log^2 n$."

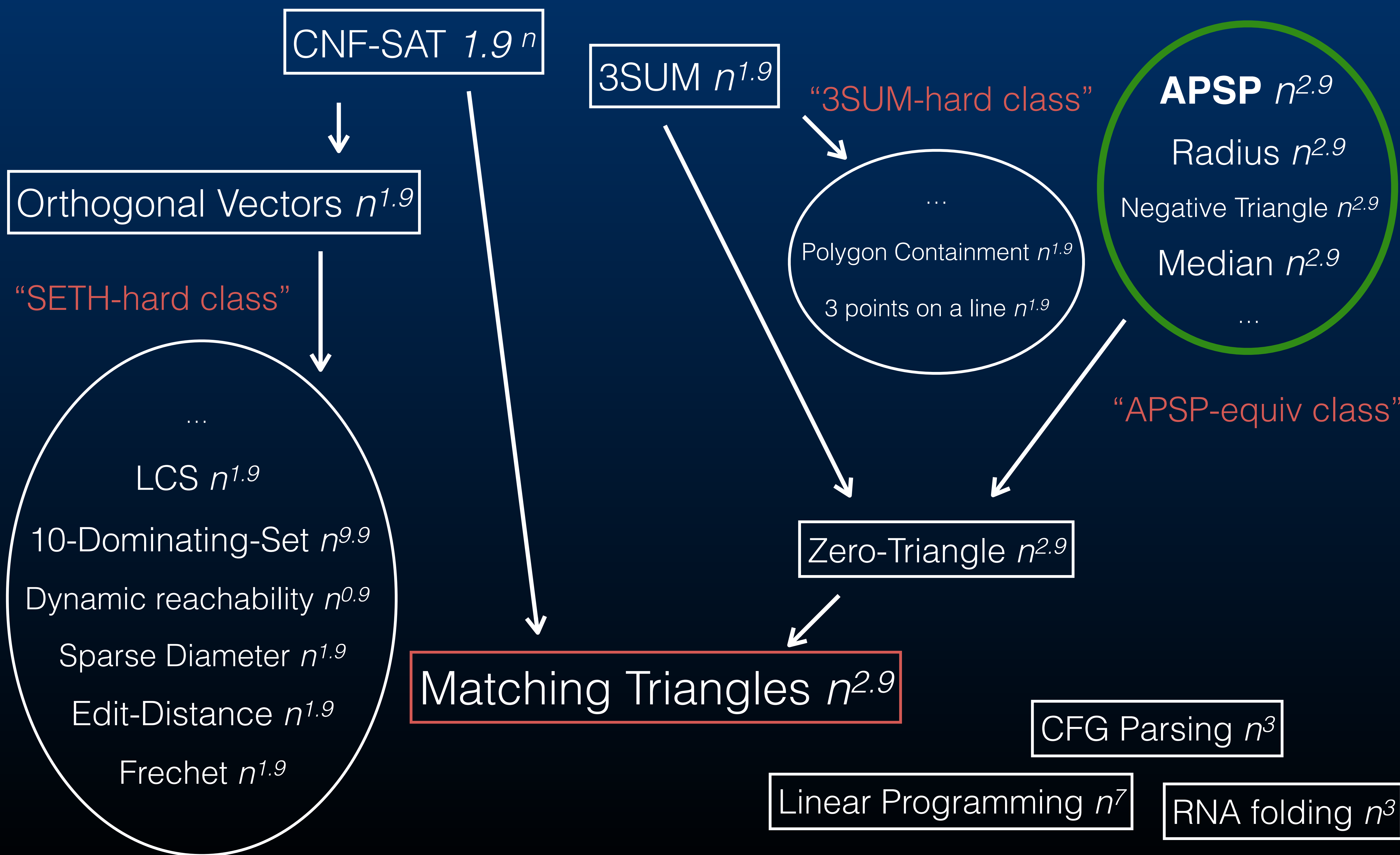
Tight lower bounds for diverse problems!

P before…



Radius n^3
CFG Parsing n^3
3SUM n^2
RNA folding n^3
Linear Programming n^7
Frechet distance n^2
LCS n^2
All Pairs Shortest Paths n^3
Local Alignment n^2
Orthogonal Vectors n^2
Edit-Distance n^2
Dynamic reachability n
Polygon containment n^2
Diameter n^3

P after…


CNF-SAT 1.9^n
3SUM $n^{1.9}$
"3SUM-hard class"
APSP $n^{2.9}$
Radius $n^{2.9}$
Negative Triangle $n^{2.9}$
Median $n^{2.9}$
…
Orthogonal Vectors $n^{1.9}$
…
Polygon Containment $n^{1.9}$
3 points on a line $n^{1.9}$
"SETH-hard class"
"APSP-equiv class"
…
LCS $n^{1.9}$
10-Dominating-Set $n^{9.9}$
Dynamic reachability $n^{0.9}$
Sparse Diameter $n^{1.9}$
Edit-Distance $n^{1.9}$
Frechet $n^{1.9}$
Zero-Triangle $n^{2.9}$
Matching Triangles $n^{2.9}$
CFG Parsing n^3
Linear Programming n^7
RNA folding n^3

Many intriguing questions remain!

➡ What’s next for “Hardness in P”?

- Better understanding of the conjectures

“strengthening the foundations of this project”

Find more reasons to believe the conjectures

"If 3SUM is in $n^{1.9}$ time, then…"

"If CNF-SAT is in 1.9^n time, then…"

Alternatively, replacing the conjectures with more plausible ones.
[A-VW-Yu STOC 15'] tomorrow!

Find more reasons to disbelieve the conjectures?

Very interesting progress on 3SUM.
[Chan-Lewenstein STOC 15'] tomorrow!

<u>Find connections between the conjectures</u>

" maybe all $O(n^2)$ problems are equivalent under subquadratic reductions"

?? ??

OVP ⇄ 3SUM ⇄ APSP

big open questions.

<u>Find barriers for relating them?</u>

➡ What's next for "Hardness in P"?

- Better understanding of the conjectures
- Classifying more problems

"If we're stuck with a bound, we should know why."

To classify all of P, new conjectures might be needed.

Unclassified problems:

Maximum Matching

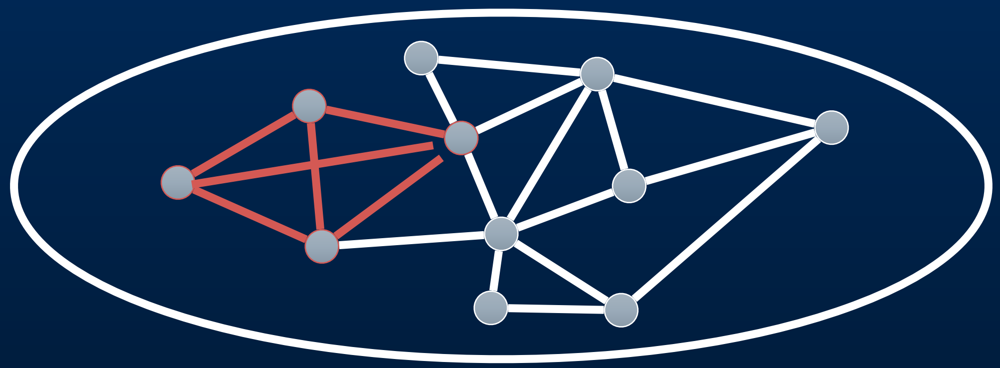
Linear Programming

Can we write a small LP for Orthogonal Vectors or 3SUM?

Recently: a new class based on k-Clique.

k-Clique

Given a graph on n nodes, are there k that form a clique?



Best combinatorial algorithm: $\sim O(n^k)$

[Nesetril - Poljak 85']: $O(n^{\omega k/3}) = O(n^{0.79k})$

[A-Backurs-VW 15']

If these algorithms are optimal, we can show very nice lower bounds in P!

[A-Backurs-VW Arxiv 15'] k-Clique based lower bounds

A very basic CS problem

CFG Parsing

Input: Context Free Grammar G and a string w of length *n*

G

S → A B
A → C B
A → A C
B → 0
C → 1

w = 10101101101

Output: Can G derive w? $S \to AB \to CBB \to \ldots \to w$

CYK, Earley's ~$O(n^3)$ Valiant's Parser $O(n^\omega)$

Theorem: faster algorithms imply faster k-Clique!

[A-Backurs-VW Arxiv 15'] k-Clique based lower bounds

An important problem in Computational Biology

RNA Folding

<u>Input</u>: A sequence in $\{A,C,G,T\}^n$.

Matches:
A - T
C - G

<u>Output</u>: Maximum number of non-crossing matches

Best Algorithms are $\sim O(n^3)$

One reason to believe the new "k-Clique Conjecture":

[Williams ICALP 04']

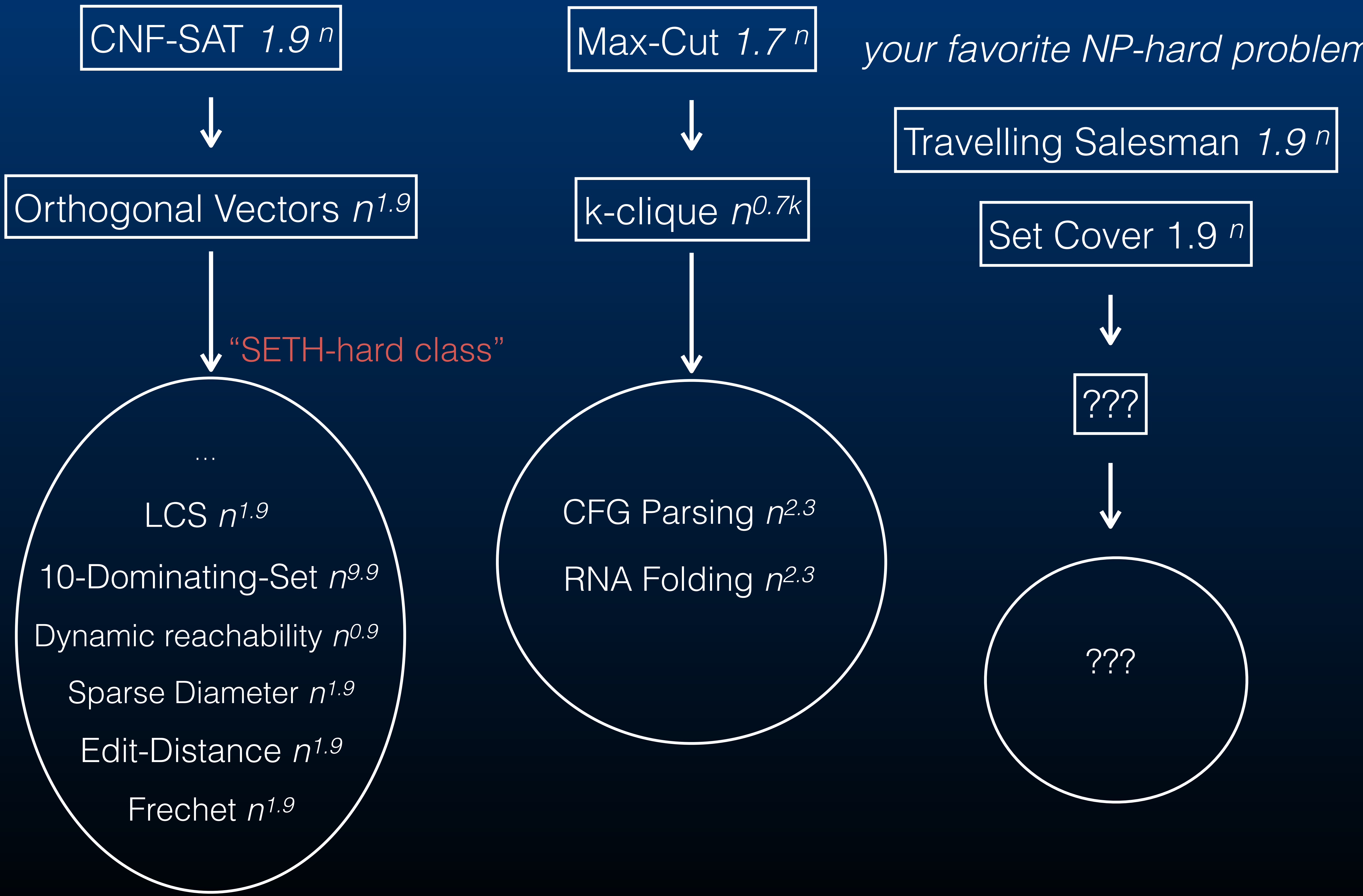
Faster k-Clique implies faster Max-Cut

Best Exact Algs for Max-Cut: $2^{\omega n/3} < 1.73^{n}$

➡ What's next for "Hardness in P"?

- Better understanding of the conjectures
- Classifying more problems
- More connections to Exact Algorithms

"More opportunities to say new things about fundamental problems"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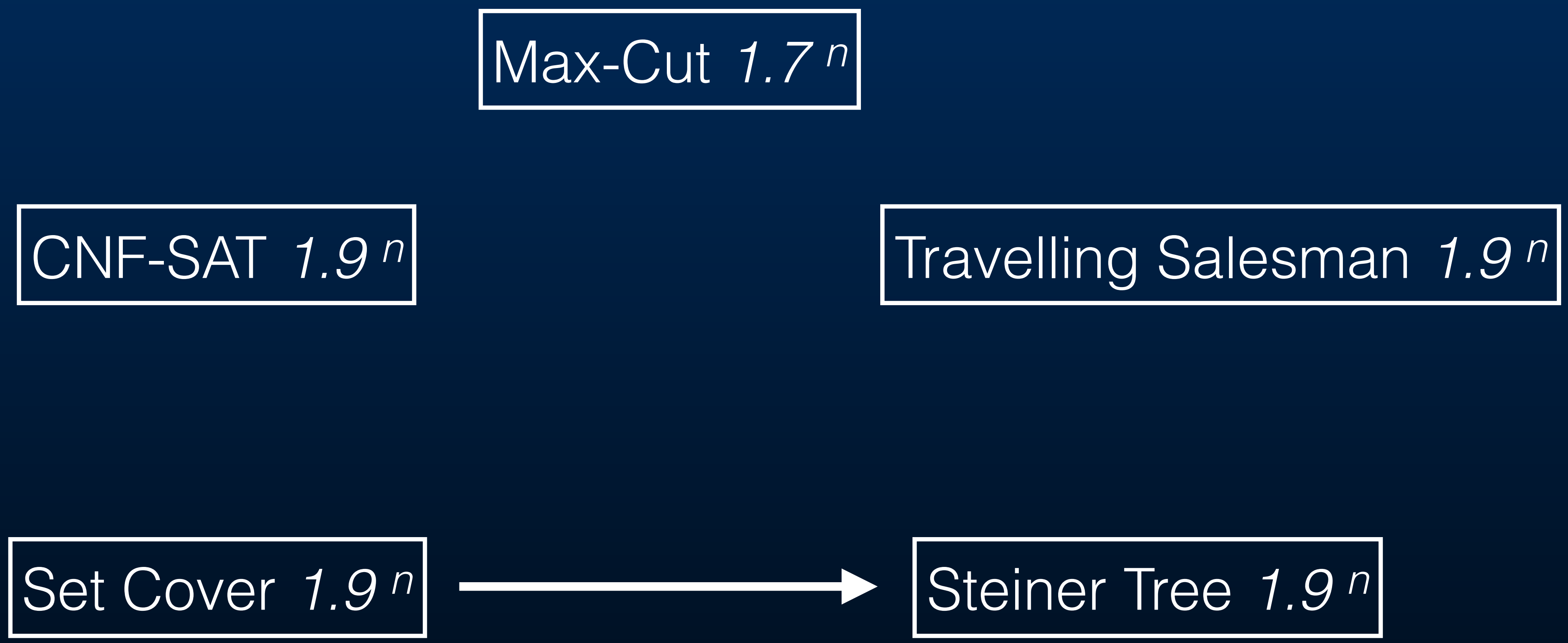
Sparse Diameter n^1.9
Edit-Distance n^1.9
Frechet n^1.9
Max-Cut 1.7 ^n
k-clique n^0.7k
CFG Parsing n^2.3
RNA Folding n^2.3
your favorite NP-hard problem
Travelling Salesman 1.9 ^n
Set Cover 1.9 ^n
???
???

Tight Reductions within NP-hard problems?



[Cygan-Dell- Lokshtanov-Marx-Nederlof-Okamoto-Paturi-Saurabh-Wahlstrom CCC 12']

➡ What’s next for “Hardness in P”?

- Better understanding of the conjectures
- Classifying more problems
- More connections to Exact Algorithms
- Fixed Parameter Tractability in P

“Apply the insights to P”

Parameterized Complexity:

Solve NP hard problems in $f(k)\, n^c$ time
on inputs of size n and some natural parameter k

[Giannopoulou - Mertzios - Niedermeier 15’] [A-VW-Wang 15’]

We should study Fixed Parameter Tractable algorithms
even for problems in P!

Fixed Parameter Subquadratic

[A-VW-Wang 15']

Case study: **Diameter** in sparse graphs.

No subquadratic algorithm under SETH.

k = treewidth of G

Upper bound:

$2^{O(k \log k)} n^{1+o(1)}$

"Lower bound":

$2^{o(k)} n^{1.9}$

refutes SETH

(the dependence on k is nearly tight!)

➡ What's next for "Hardness in P"?

- Better understanding of the conjectures
- Classifying more problems
- More connections to Exact Algorithms
- Fixed Parameter Tractability in P
- Hardness of approximation

"Even more relevant lower bounds"

<u>Best Approximation for Edit Distance in Subquadratic time?</u>

Linear time approximations and heuristics are being used in practice

*Near-linear time **polylog** approximation is known*
[Andoni - Krauthgamer - Onak FOCS 10']

PCP Theorem in P?

➡ What’s next for “Hardness in P”?

- Better understanding of the conjectures
- Classifying more problems
- More connections to Exact Algorithms
- Fixed Parameter Tractability in P
- Hardness of approximation

Many more exciting questions:

- Barriers for shaving more log factors?
- Average case hardness?
- Quantum Algorithms?
- Space Complexity?

Thanks for attending!