CS267 HOMEWORK 1 **Due: 11:59pm, Thu, Oct. 13**
Submit by email to `cs267stanford@gmail.com` Handed out: October 3, 2016

Instructions: Everyone needs to submit their own write-up. If you work together with other students, indicate their names on your write-up.

All graphs in this problem set are undirected, simple and unweighted.

Problem 1

Let P_3 be the path on 3 nodes. We say that an undirected graph $G = (V, E)$ contains an induced P_3 if there exist $u, v, w \in V$ such that $(u, v), (v, w) \in E$ but $(u, w) \notin E$.

Design an $O(m+n)$ time algorithm that finds an induced P_3 in any m-edge, n-node graph G, or determines that G does not contain an induced P_3.

Hint: A graph contains a P_3 if and only if it is **not** a node-disjoint union of cliques (of possibly different sizes; a single node is considered a clique of size 1; you may want to prove this).

Problem 2

In class we showed that a 4-cycle (if one exists) in an n-node graph can be found in $O(n^2)$ time. Give a similar algorithm that can find a 4-cycle in an m-edge n-node graph in $O(m\sqrt{n})$ time. (Hint: use the high degree-low degree technique)

BONUS: Give an algorithm for finding a 4-cycle running in $O(m^{4/3})$ time.

Problem 3

1. Show that exact girth of an n-node, m-edge graph can be found in $O(nm)$ time (Hint: your algorithm may look similar to Itai and Rodeh's algorithm). Explain the worst-case behavior that would cause your algorithm to require $O(nm)$ time, rather than $O(n^2)$.

2. Let G be an n-node graph such that for some integer $g \geq 1$, the minimum degree of G is $> \lceil n^{1/g} \rceil$. Prove that G has girth at most $2g$.

3. Let $k \geq 1$ be a given integer and suppose G is an n-node regular graph (i.e. the degrees of all vertices in G are the same). Show that one can either find in $O(m + n)$ time a cycle of length at most $2k$, or in $O(n^{2+1/k})$ time one can compute the exact girth. (Hint: use (a) and (b).)