An $O(n^{1.5})$ Deterministic Gossiping Algorithm for Radio Networks

Ying Xu
Dept of Computer Science, Peking University, Beijing, 100871, China
Dept of Computer Science, City University, Tat Chee Road, Kowloon, Hong Kong
Email address: xuying@cs.cityu.edu.hk*

Abstract

We consider the problem of distributed gossiping in radio networks of unknown topology. For radio networks of size n and diameter D, we present an adaptive deterministic gossiping algorithm of time $O(\sqrt{D}n + nlog^2n)$, or $O(n^{1.5})$. This algorithm is a tuned version of the fastest previously known gossiping algorithm due to Gasieniec and Lingas ([1]), and improves the time complexity by a poly-logarithmic factor.

Key Words. gossiping, radio network, deterministic algorithm

*This research is supported by CERG Grant [CityU1070/02E] of Hong Kong

1 Introduction

A *radio network* is a collection of distributed transmitter-receiver devices, each of which can reach a given subset of devices. Such a radio network can be modelled as a directed graph, with nodes in the graph representing devices and directed edges representing their reachability relationships. All these nodes send out messages in synchronous time slots, and the message reaches all the neighbors in the same time slot. However, if two or more messages reach a node in the same time slot, none of the messages could be received, and the node cannot distinguish this case from the case that no message comes. See [2] for a detailed description of the model.

One of the most extensive studied problems in radio networks is the *gossiping problem* (see e.g. [2, 1, 3]), in which every node is initially given a different message that needs to be distributed to all other nodes. We assume that the network is strongly connected for gossiping to be feasible. In this paper, we consider the model where initially the nodes have no knowledge about its topology. For gossiping in unknown radio networks of size n and diameter D, we propose an $O(n^{1.5})$ ($O(\sqrt{D}n + nlog^2n)$ indeed) deterministic algorithm. The best previously known result is $\tilde{O}(n^{1.5})$. [2] gave a non-constructive algorithm working in time $O(n^{1.5}log^2n)$, and later Indyk showed in [4] an alternative constructive solution with similar complexity $\tilde{O}(n^{1.5})$. In [1], an adaptive algorithm of time $O(\sqrt{D}nlogn)$ was proposed. For the randomized algorithm, [3] gave an algorithm with expected running time $O(nlog^4n)$, and then [5] improved it to $O(nlog^3n)$. Our algorithm is a modified version of the algorithm by Gasieniec and Lingas ([1]).

2 Preliminaries

2.1 Broadcasting complexity

The *Broadcasting problem* is a fundamental problem in radio networks, and is often used as a sub-procedure in gossiping algorithm. In the broadcasting problem, one distinguished node has a message that needs to be distributed to all other nodes. Denote by $B(n)$ the time complexity of broadcasting in unknown radio network of size n. [2] presented a non-constructive broadcasting algorithm working in time $O(nlog^2n)$, and later [4] gave a constructive version with similar complexity $\tilde{O}(n)$.

2.2 Round Robin algorithm

Round Robin is the most commonly known communication procedure for radio networks. One Round Robin round takes n time slots; in time slot i, node i sends a message (note it can compress its whole knowledge into one message). For both broadcasting and gossiping problems, Round Robin completes in D rounds, where D is the diameter of the network, i.e. the maximum length of the shortest directed path between any two nodes.

2.3 An $O(n^{1.5}logn)$ time gossiping algorithm

[1] gave an $O(\sqrt{D}nlogn) = O(n^{1.5}logn)$ time gossiping algorithm. It works as follows. First, run r Round Robin rounds so that every node v collects information about its in-neighborhood of radius r, denoted by $N_r^-(v)$. Then choose an arbitrary node as the central node, say node 1, which gradually builds up the knowledge of its neighborhood and schedules transmission of the network. Initially, node 1 has knowledge of $N_r^-(1)$ (after Round Robin). The algorithm then proceeds in $\lceil D/r \rceil$ phases. In phase i, node 1 distributes $N_{ir}^-(1)$ to all nodes by broadcasting. Then all nodes v in $N_{ir}^-(1)$ send back to 1 information about their in-neighborhood $N_r^-(v)$, so that node 1 now knows $N_{(i+1)r}^-(1)$, the in-neighborhood of radius $(i+1)r$. In this way, node 1 gains the knowledge of its in-neighborhood by radius r in each phase. After $\lceil D/r \rceil$ such phases, node 1 has the knowledge of the whole network. Note that in phase i, when the nodes send back information to 1, they already know the topology of $N_{ir}^-(1)$, so this sub-step can accomplish in only linear time. Therefore, the total time of above algorithm is $O(rn + B(n) * D/r)$. [1] claimed an $\tilde{O}(\sqrt{D}n)$ bound by assigning $r = \sqrt{D}$ and $B(n) = O(nlog^2n)$. Let $r = \sqrt{D}logn$, it can achieve $O(\sqrt{D}nlogn)$.

3 Our $O(n^{1.5})$ time algorithm

Like the algorithm in [1], our new algorithm is adaptive, i.e. scheduling the transmissions according to the network topology information collected so far. The basic idea of the improvement is to also collect out-paths of node 1 and use this information to reduce the time for broadcasting $N_{ir}^-(1)$ from 1 to other nodes.

First of all, node 1 initiates a broadcast. During the broadcast, the nodes attach their labels to the message when pass it on, so that after broadcasting every node v knows a directed path from node 1 to itself, denoted $p(1, v)$. (If more than one path is detected by v, choose any one.) The remaining steps are similar to [1], but when node v sends back a message to node 1, it sends $p(1, u)$ for all $u \in N_r^-(v)$ as well as $N_r^-(v)$; and when node 1 sends $N_{ir}^-(1)$ to other nodes, it can arrange the transmission order according to those $p(1, v)$'s, instead of broadcasting as it does in [1]. Denote by $T_d^-(1)$ a directed tree rooted at node 1 which contains and only contains nodes in $N_d^-(1)$ with all edges directing from bottom to root, and by $T_d^+(1)$ a directed tree rooted at 1 which contains all nodes in $N_d^-(1)$ with all edges directing from root to bottom. Note that $T_d^+(1)$ may contain internal nodes outside $N_d^-(1)$. The complete algorithm is given below. We assume that the size of the network is known. For simplicity, we also assume that we know the network diameter D.

The latter assumption can be dropped by applying standard doubling technique ([2]).

Algorithm

1. Node 1 initiates a deterministic broadcast. Each node v collects information $p(1, v)$;

2. Run Round Robin for r rounds. Each node v collects information about $N_r^-(v)$, as well as $p(1, u)$ for any $u \in N_r^-(v)$;

3. For $i = 1, \ldots, \lceil D/r \rceil$ do

(a) Node 1 constructs $T_{ir}^-(1)$ and $T_{ir}^+(1)$;

(b) Node 1 distributes $T_{ir}^-(1)$ and $T_{ir}^+(1)$ to all nodes in $N_{ir}^-(1)$;

(c) Each node in $N_{ir}^-(1)$ sends $N_r^-(v)$ and $p(1, u)$ for any $u \in N_r^-(v)$, as well as its initial message, back to 1.

4. Node 1 distributes all messages it collected to all nodes.

Theorem 1 *The algorithm completes gossiping in time* $O(\sqrt{D}n + nlog^2n)$.

Proof. First we show the algorithm correctly completes gossiping. The invariant is that at the beginning of iteration i in step3, node 1 possesses enough information to construct $T_{ir}^-(1)$ and $T_{ir}^+(1)$. For every node $v \in N_{(i+1)r}^-(1)$, but $\notin N_{ir}^-(1)$, there must be some node $u \in N_{ir}^-(1)$ and $v \in N_r^-(u)$ who will send $p(1, u)$ in substep(c) of iteration i. So after iteration i, node 1 knows at least one directed path from itself out to v and one path from v to itself. Trimming some edges from the union of such in-paths and $T_{ir}^-(1)$, we can get a tree $T_{(i+1)r}^-(1)$. $T_{(i+1)r}^+(1)$ is obtained by trimming some edges in the union of $p(1, v)$'s for all $v \in N_{(i+1)r}^-(1)$. In the last iteration of step 3, all nodes in $N_D^-(1)$, i.e. the whole network, send their messages to 1. And in step 4, node 1 distributes all messages to all nodes. Therefore, gossiping is completed.

Next we analyze the time complexity of the algorithm.

Step1 takes time $B(n) = O(nlog^2n)$;

Step2 takes time rn, since one Round Robin round takes time n.

Step3. Since we are only interested in communication complexity, the cost of Substep(a) is null. In substep(b), the nodes in $T_{ir}^+(1)$ transmit according to the pre-order of $T_{ir}^+(1)$, one node in a time slot. So the time complexity of substep(b) in one iteration is bounded by n. (Note that a node transmits in iteration i only after it receives $T_{ir}^+(1)$, so it knows exactly when it should transmission.) In substep(c), the nodes in $N_{ir}^-(1)$ transmit according to the post-order of $T_{ir}^-(1)$, so the time complexity of substep(c) in one iteration is also bounded by n.

Step4 takes time n. The argument is similar to that of 3.(b).

Therefore, the total time of the algorithm is $O(nlog^2n + rn + nD/r)$. Let $r = \sqrt{D}$, the time is $O(\sqrt{D}n + nlog^2n)$, which completes the proof. □

Since the diameter of a (strongly connected) directed graph is at most n, we have the following corollary.

Corollary 2 *The algorithm completes gossiping in time $O(n^{1.5})$.*

4 Final Remarks

The running time of our gossiping algorithm is combined by two parts, one broadcasting followed by consecutive phases of communication controlled by a central node. Unlike all the previous gossiping algorithms, our algorithm does not require an extensive use of the broadcasting procedure. When the network diameter D is large ($D = O(n^{\alpha})$), the consecutive communication phases dominates the running time, and we can use the $\tilde{O}(n)$ constructive algorithm for broadcasting. So the $O(n^{1.5})$ upper bound is established with a constructive algorithm. On the other hand, for small values of D ($D < log^4 n$), the time complexity is dominated by broadcasting while gossiping overhead is null. It has not been known whether gossiping is harder than broadcasting in the radio network model. Our result identifies an interesting subclass of networks which have the same gossiping and broadcasting complexity.

Acknowledgements We are grateful to an anonymous referee for pointing out some interesting consequences of the algorithm.

References

[1] L. Gasieniec and A. Lingas, On adaptive deterministic gossiping in ad hoc radio networks, *Inform. Process. Lett.*, **2**(2002), 89-93.

[2] M. Chrobak, L. Gasieniec, and W. Rytter, Fast broadcasting and gossiping in radio networks, in *Proc. 41st IEEE Symp. on Found. of Computer Science,* 2000, pp.575-581.

[3] M. Chrobak, L. Gasieniec, and W. Rytter, A randomized algorithm for gossiping in radio networks, in *Proc. 7th Annual Int. Computing and Combinatorics Conference,* 2001, pp.483-492.

[4] P. Indyk, Explicit constructions of selectors and related combinatorial structures with applications, in *Proc. 13th ACM-SIAM Symp. on Discrete Algorithms,* 2002, pp.697-703.

[5] D. Liu and M. Prabhakaran, On randomized broadcasting and gossiping in radio networks, in *Proc. 8th Annual Int. Computing and Combinatorics Conference,* 2002, pp.340-349.