CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2009

Assignment #2 - Due date: Tuesday, 4/21/09

Problem 1. [10 points] Solve parts (c) and (d) of Exercise 3.2.6 on page 108 of the textbook. (Look at the solutions for parts (a) and (b) for some hints.)

Problem 2. [20 points] Provide regular expressions for the following languages over the alphabet $\Sigma = \{0, 1\}$. Provide a brief explanation as to why your regular expressions generate the given languages.

a). The set of all strings with at most one pair of consecutive 1’s.
b). The set of all strings not containing 010 as a substring.

Problem 3. [15 points] Consider the DFA $M = (Q, \Sigma, \delta, q_0, F)$ with $Q = \{1, 2, 3, 4\}, \Sigma = \{a, b, c\}, q_0 = 1, F = \{4\}$, and $\delta$ as defined in the following transition diagram.

(a). [10 points] Recall the construction given in class for converting a DFA into a regular expression. Given a DFA, we had defined the notion of a regular expression $R_{ij}^{(k)}$ for the language $L_{ij}^{(k)}$ consisting of all strings that go from state $q_i$ to state $q_j$ without visiting a state numbered larger than $k$ along the way.

For the DFA $M$ given above, specify the following regular expressions. You do not need to justify your answer and are free to use any method to determine the answer (including "reasoning it out").

1) $R_{35}^{(0)}$
2) $R_{12}^{(0)}$
3) $R_{12}^{(4)}$
4) $R_{11}^{(0)}$
5) $R_{11}^{(4)}$

(b). [5 points] Provide a regular expression for $L(M)$.

**Problem 4.** [10 points] Solve Exercise 4.1.1(f) on page 131 of the textbook.

**Problem 5.** [10 points] Solve Exercise 4.1.2(b) on page 132 of the textbook. (Hint: See the solution for Exercise 4.1.2(a).)

**Problem 6.** [35 points]
Consider the following language $L$ over the alphabet $\Sigma = \{a, b, c\}$.

$$L = \{a^kb^lc^m \mid k, \ell, m \geq 0 \text{ and if } k = 1 \text{ then } \ell = m\}.$$ 

It is known that even though $L$ is non-regular, it still satisfies the conditions of the pumping lemma for regular languages.

(a). [15 points] Assume that the pumping lemma constant $n$ is sufficiently large, say $n > 4$. Consider the following three strings in $L$:

$$b^\ell c^m, ab^n c^m, a^2b^\ell c^m, \text{ and } a^3b^\ell c^m$$

where $\ell > n$ and $m > n$. For each of these strings, show that there does exist a way of decomposing them into $xyz$ such that the decomposition satisfies the pumping lemma, in that: (1) $y \neq c$; (2) $|xy| \leq n$; and, (3) for all $i \geq 0$, $xy^iz \in L$.

(b). [5 points] In general, it can be shown that a decomposition consistent with the pumping lemma can be obtained for any string $w \in L$ which is longer than $n$. This means that although $L$ is non-regular, we cannot use the pumping lemma to show that $L$ is non-regular. Explain why this does not contradict the pumping lemma.

(c). [15 points] Show that the language $L$ is non-regular. (Hint: Clearly, by part (a), you cannot directly use the pumping lemma. Consider using closure properties of regular languages to convert $L$ into a language where you can indeed use the pumping lemma.)

**Reading Assignment:**

In Chapter 3, we have covered Sections 3.1 and 3.2 in detail. But we also recommend that you read Sections 3.3 and 3.4 for background material. We have covered portions of Section 3.4 in the class.

Next, we will cover Sections 4.1, 4.2, and 4.3 in Chapter 4 of the course reader. We will only skim over the material in Section 4.4 — but we recommend that you read the main ideas in this section without getting into the technical details. Next up: Chapter 5.