Problem 1. [15 points] Solve Exercise 8.1.1(b) on page 324 of the textbook.

Problem 2. [15 points] Show that there is no program for solving the following problem: Given a program $Q$ and an input $y$, during the execution of program $Q$ on input $y$ do the two integer variables named $A$ and $B$ ever take on the same value?

Problem 3. [15 points] Consider the (deterministic) Turing machine $M$ given by

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined in it, as described below.

1. $\delta(q_0, a) = (q_0, B, R)$
2. $\delta(q_0, b) = (q_1, B, R)$
3. $\delta(q_1, b) = (q_1, B, R)$
4. $\delta(q_1, B) = (q_2, B, R)$

(a). [5 points] Specify the execution trace of $M$ on the input string $abb$.
(b). [5 points] Provide a regular expression for the language of this Turing machine.
(c). [5 points] Suppose we added the following transition to the above machine.

$$\delta(q_1, a) = (q_0, B, R)$$

Provide a regular expression for the language of the resulting Turing machine.

Problem 4. [15 points] Consider the Turing machine $M$,

$$M = (\{q_0, q_1, q_2, f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{f\})$$

whose transitions are defined below.

$$\delta(q_0, 0) = \{(q_1, 1, R)\} \quad \delta(q_1, 1) = \{(q_2, 0, L)\} \quad \delta(q_2, 1) = \{(q_0, 1, R)\} \quad \delta(q_1, B) = \{(f, B, R)\}$$

(a). [5 points] Provide the execution trace of this machine on the input 011.
(b). [5 points] Describe the language accepted by $M$.
(c). [5 points] Suppose that the transition function defined above is modified so that the transition $\delta(q_0, 0) = \{(q_1, 1, R)\}$ is replaced by the following.

$$\delta(q_0, 0) = \{(q_1, 1, R), (q_0, 0, R)\}.$$ 

Describe the language accepted by $M$. (Note that this new machine is non-deterministic.)
Problem 5. [20 points] Given the encoding of a deterministic Turing machine $M$ and its input $w$, output YES if $M$ running on input $w$ will NEVER move its tape-head leftwards. (Hint: Look at the solution to Exercise 9.3.6(a).)

Problem 6. [20 points] Solve Exercise 9.3.6(c) on page 400 of the textbook. (Hint: Look at the solution to Exercise 9.3.6(a).)

Reading Assignment: We are currently working on Chapter 8. In Section 8.5, you should read 8.5.1 but we will omit the rest in this course. Section 8.6 will not be covered in class, but it is important that you at least skim through this section.