6. Program Correctness: Strategies

Developing Inductive Assertions

Some structured techniques for developing inductive annotations for proving partial correctness. Just heuristics.

Basic Facts

Example: LinearSearch

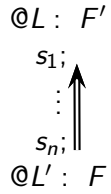
```
for
  @L : ℓ ≤ i ≤ u + 1
  (int i := ℓ; i ≤ u; i := i + 1) {
  if (a[i] = e) return true;
}
```


Example: BubbleSort

```
for
  @L1 : −1 ≤ i < |a|
  (int i := |a| − 1; i > 0; i := i − 1) {
  for
    @L2 : 0 < i < |a| ∧ 0 ≤ j ≤ i
    (int j := 0; j < i; j := j + 1) {
    if (a[j] > a[j + 1]) {
      int t := a[j];
      a[j] := a[j + 1];
      a[j + 1] := t;
    }
  }
}
```


The Precondition Method

- Given annotation @L : F
- Compute the precondition of F backward
- Find new annotation @L' : F'



Example: BinarySearch

```
@pre H?
@post ⊤
bool BinarySearch(int[] a, int ℓ, int u, int e) {
  if (ℓ > u) return false;
  else {
    @ 2 ≠ 0;                    ... basic fact
    int m := (ℓ + u) div 2;
    @ 0 ≤ m < |a|;              ... basic fact
    if (a[m] = e) return true;
    else if (a[m] < e) return BinarySearch(a, m + 1, u, e);
    else return BinarySearch(a, ℓ, m − 1, e);
  }
}
```

(·)

@pre H : ?
S_1 : `assume` $\ell \leq u$;
S_2 : $m := (\ell + u)$ `div` 2;
@ F : $0 \leq m < |a|$

Compute

$$
\begin{aligned}
& \mathsf{wp}(F,\ S_1; S_2) \\
& \Leftrightarrow\ \mathsf{wp}(\mathsf{wp}(F,\ m := (\ell + u)\ \texttt{div}\ 2),\ S_1) \\
& \Leftrightarrow\ \mathsf{wp}(F\{m\ \mapsto\ (\ell + u)\ \texttt{div}\ 2\},\ S_1) \\
& \Leftrightarrow\ \mathsf{wp}(F\{m\ \mapsto\ (\ell + u)\ \texttt{div}\ 2\},\ \texttt{assume}\ \ell \leq u) \\
& \Leftrightarrow\ \ell \leq u\ \rightarrow\ F\{m\ \mapsto\ (\ell + u)\ \texttt{div}\ 2\} \\
& \Leftrightarrow\ \ell \leq u\ \rightarrow\ 0 \leq (\ell + u)\ \texttt{div}\ 2 < |a| \\
& \Leftarrow\ 0 \leq \ell\ \wedge\ u < |a|
\end{aligned}
$$

@pre H : $0 \leq \ell\ \wedge\ u < |a|$

guaranteed

$$0 \leq \ell\ \wedge\ u < |a|\ \rightarrow\ \mathsf{wp}(F,\ S_1; S_2)$$

is $T_{\mathbb{Z}}$-valid. The runtime assertion

$$0 \leq m < |a|$$

holds in every execution of BinarySearch in which the precondition

@pre $0 \leq \ell\ \wedge\ u < |a|$

is satisfied.