THE CALCULUS OF COMPUTATION:
Decision Procedures with
Applications to Verification

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8. Quantifier-Free Linear Arithmetic

Decision Procedures for Quantifier-free Fragments

For theory T with signature Σ and axioms Σ-formulae of form
$\forall x_1, \ldots, x_n.\ F[x_1, \ldots, x_n]$

Decide if
$F[x_1, \ldots, x_n]$ or $\exists x_1, \ldots, x_n.\ F[x_1, \ldots, x_n]$ is T-satisfiable

[Decide if
$F[x_1, \ldots, x_n]$ or $\forall x_1, \ldots, x_n.\ F[x_1, \ldots, x_n]$ is T-valid]

where F is quantifier-free and $\text{free}(F) = \{x_1, \ldots, x_n\}$

Note: no quantifier alternations

We consider only conjunctive quantifier-free Σ-formulae, i.e., conjunctions of Σ-literals (Σ-atoms or negations of Σ-atoms).
For given arbitrary quantifier-free Σ-formula F, convert it into DNF Σ-formula
$F_1 \vee \ldots \vee F_k$
where each F_i conjunctive.
F is T-satisfiable iff at least one F_i is T-satisfiable.