

<p style="text-align: center;">MATH 108: Introduction to Combinatorics, Winter 2016 HOMEWORK 4 Due Monday, February 8</p>
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You should solve the homework on your own. Don't use any books or the internet.

Problem 1. [Knuth p. 308, problem 6]

Prove $a_j = b_j \oplus b_{j+1}$, the basic formula for the Gray code. Then, using this show that if $g(k) = (\dots, a_2, a_1, a_0)$ in base 2 then $b_j = a_j + a_{j+1} + \dots$ where all addition is mod 2 and the sum is actually finite.

Problem 2. [Knuth p. 315, problem 67]

Explain how to traverse all binary n -tuples in such a way that each step changes n or $n - 1$ bits, alternately.

Problem 3. Find a sequence of 52 zeros and ones such that if a window of length 6 is run along (including around the corner) all the binary six-tuples that appear are distinct. Show how to arrange an ordinary deck of 52 cards so that the red/black pattern matches your order.

Problem 4. Consider the following "greedy rule" for constructing a binary de Bruijn sequence of length 2^k : start with k zeros, and then always add a 1 if this doesn't duplicate a previous pattern, or else add a 0. Try this rule out when $k = 5$ and check that it works. Prove that it works for all k .